GS2017- Mathematics: Selection Procedure

Entrance examination for the TIFR Graduate Programs 2017 in Mathematics (PhD and Integrated PhD at TIFR, Mumbai, PhD and Integrated PhD at CAM, Bengaluru, and PhD at ICTS, Bengaluru) will comprise of a written test and an interview, as follows:

Written test

The written test will be of total 3 hours duration and it has two parts, Part I and Part II. In Part I there will be 30 true or false questions. A correct answer gets 2 points, a wrong answer gets -1 point, non answer gets 0 points. The answer sheets for Part I will be collected at the end of 1 1/2 hours.

Only Part I will be graded for all applicants. Based on a suitable cutoff, approximately around 100 top candidates will be chosen for the next step of evaluation. The answer booklets for Part II for will be graded only for these chosen candidates.

Part II consists of 10 questions, and the solution to the questions have to be written. Credit will be given for the answers, and partial credit will be given for partially correct answers.

Syllabus for written test

The screening test is mainly based on mathematics covered in a reasonable B.Sc. course. The interview need not be confined to this.

Algebra: Definitions and examples of group (finite and infinite, commutative and non-commutative), cyclic groups, subgroups, homomorphisms, quotients. Definitions and example of rings and fields. Basic facts about finite dimensional vector spaces, matrices, determinants, and ranks of linear transformations. Integers and their basic properties. Polynomials with real or complex coefficients in 1 variable.

Analysis: Basic facts about real and complex numbers, convergence of sequences and series of real and complex numbers, continuity, differentiability and Riemann integration of real valued functions defined on an interval (finite or infinite), elementary functions (polynomial functions, rational functions, exponential and log, trigonometric functions).

Geometry/Topology: Elementary geometric properties of common shapes and figures in 2 and 3 dimensional Euclidean spaces (e.g. triangles, circles, discs, spheres, etc.). Plane analytic geometry (= coordinate geometry) and trigonometry. Definition and basic properties of metric spaces, examples of subset Euclidean spaces (of any dimension), connectedness, compactness. Convergence in metric spaces, continuity of functions between metric spaces.

General: Pigeon-hole principle (box principle), induction, elementary properties of divisibility, elementary combinatorics (permutations and combinations, binomial coefficients), elementary reasoning with graphs.

Selection Interviews

Based on the combined performance, a further shortlist will be made, for each program. The listed candidates will be called for final selection interviews for various programs of TIFR in different campuses of TIFR.

Sample Questions

Part I

Answer whether the following statements are True or False. Mark your answer on the machine checkable answer sheet that is provided.

Note: $+2$ *marks for a correct answer,* -1 *mark (negative marks) for a wrong answer,* 0 *marks for not answering.*

1. If A and B are 3×3 matrices and A is invertible, then there exists an integer n such that $A + nB$ is invertible.

2. Let P be a degree 3 polynomial with complex coefficients such that the constant term is 2010. Then P has a root α with $|\alpha| > 10$.

3. The symmetric group S_5 consisting of permutations on 5 symbols has an element of order 6.

4. Suppose $f_n(x)$ is a sequence of continuous functions on the closed interval $[0,1]$ converging to 0 pointwise. Then the integral
$$\int_0^1 f_n(x)dx$$
converges to 0.

5. There are n homomorphisms from the group $\mathbb{Z}/n\mathbb{Z}$ to the additive group of rationals $\mathbb{Q}$.

6. A bounded continuous function on $\mathbb{R}$ is uniformly continuous.

Part II

Write your solutions in the answer booklet provided. All questions carry equal marks. There are no negative marks, and partial credit will be given for partial solutions.

1. Let F be a field. Prove that the polynomial ring $F[x]$ in one variable over F has infinitely many prime ideals.

2. Let $c_1, \ldots, c_r$ be distinct real numbers. Show that the functions $e^{c_1 t}, \ldots, e^{c_r t}$ on $\mathbb{R}$ are linearly independent.

3. Let $f_n : [a,b] \to \mathbb{R}$ be a sequence of continuous functions converging uniformly to a function f.

   (a) Show that if each f_n has a zero then f has a zero.

   (b) Give an example of a sequence of functions on $[0, +\infty)$ converging uniformly such that each function in the sequence has a zero but the limit function is nowhere zero.