Chap-1 - Sets, Relations and Functions

Formulae Chart

- If A is a set with $n(A) = m$ then,
  a. Numbe of subsets of $A = 2^m$
  b. Number of proper subsets $= 2^m - 1$
- If A and B are finite sets then
  $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
- If A, B, C are finite sets, then
  $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$.
- If $n(A) = n$
  a. Number of identity relation in set $A = 1$.
  b. Number of reflexive relation in set $A = 2^{n^2 - n}$.
- $n(A) = m$, $n(B) = n$
  a. Number of relations from A to $B = 2^{mn}$
  b. Number of functions from A to $B = n^m$
  c. Number of one-one functions
     (i) If $m > n$, then number of one-one function = 0
     (ii) If $n \geq m$, then number of one-one function $= {}^nP_m$

Chap-02 - Trigonometric Functions

Formulae Chart

Summation/Continued Product

$$\sin\theta + \sin(\theta+\phi) + \sin(\theta+2\phi) + \ldots\ldots \text{ to } n \text{ terms} = \frac{\sin\frac{n\phi}{2}}{\sin\frac{\phi}{2}}\sin\left(\theta + \frac{n-1}{2}\phi\right) \quad (\phi \neq 2n\pi, n \in Z)$$

$$= \frac{\sin n\left(\frac{\text{difference}}{2}\right)}{\sin\frac{\text{difference}}{2}}\sin\left\{\frac{\text{first angle} + \text{last angle}}{2}\right\}$$

$$\cos\theta + \cos(\theta+\phi) + \cos(\theta+2\phi) + \ldots\ldots \text{ to } n \text{ terms} = \frac{\sin\frac{n\phi}{2}}{\sin\frac{\phi}{2}}\cos\left(\theta + \frac{n-1}{2}\phi\right) \quad (\phi \neq 2n\pi, n \in Z)$$

$$= \frac{\sin n\left(\frac{\text{difference}}{2}\right)}{\sin\frac{\text{difference}}{2}}\cos\left\{\frac{\text{first angle} + \text{last angle}}{2}\right\}$$

$$\cos A\cos 2A\cos 2^2A\cos 2^3A\ldots\ldots\cos 2^{n-1}A = \frac{1}{2^n\sin A}(\sin 2^nA) \quad (A \neq n\pi, n \in Z)$$

$\sqrt{2+\sqrt{2+\sqrt{2+\ldots\ldots+\sqrt{2+2\cos 2^nA}}}} = 2\cos A, \forall n \in N$, where there are n square root sign on L.H.S.

$\tan A + 2\tan 2A + 2^2\tan 2^2A + \ldots\ldots + 2^n\tan 2^nA + 2^{n+1}\cot 2^{n+1}A = \cot A, \forall n \in N$.

- The solution of trigonometric equations of the form $a\sin\theta \pm b\cos\theta = c$ exists iff $\left|\frac{c}{\sqrt{a^2+b^2}}\right| \leq 1$.

Napier's Analogy or Tangent Rule

In any ΔABC,

$\tan\frac{A-B}{2} = \frac{a-b}{a+b}\cot\frac{C}{2}$

$\tan\frac{B-C}{2} = \frac{b-c}{b+c}\cot\frac{A}{2}$

$\tan\frac{C-A}{2} = \frac{c-a}{c+a}\cot\frac{B}{2}$

Half-angle formulae

In any ΔABC,

$\sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$, $\sin\frac{B}{2} = \sqrt{\frac{(s-c)(s-a)}{ca}}$, $\sin\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}}$

$\cos\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$, $\cos\frac{B}{2} = \sqrt{\frac{s(s-b)}{ca}}$, $\cos\frac{C}{2} = \sqrt{\frac{s(s-c)}{ab}}$

$\tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} = \frac{(s-b)(s-c)}{\Delta}$, $\tan\frac{B}{2} = \sqrt{\frac{(s-c)(s-a)}{s(s-b)}} = \frac{(s-c)(s-a)}{\Delta}$

$\tan\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}} = \frac{(s-a)(s-b)}{\Delta}$

$\cot\frac{A}{2} = \sqrt{\frac{s(s-a)}{(s-b)(s-c)}} = \frac{s(s-a)}{\Delta}$, $\cot\frac{B}{2} = \sqrt{\frac{s(s-b)}{(s-c)(s-a)}}$

$= \frac{s(s-b)}{\Delta}$, $\cot\frac{C}{2} = \sqrt{\frac{s(s-c)}{(s-a)(s-b)}} = \frac{s(s-c)}{\Delta}$

Chap-03 - Inverse Trigonometric Functions

Formulae Chart

Inverse Trigonometric Function

1.

A	B
(i) $\sin^{-1}\sin\theta = \theta$, for all $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	(i) $\sin(\sin^{-1}x) = x$, $x \in [-1, 1]$
(ii) $\cos^{-1}\cos\theta = \theta$, for all $\theta \in [0, \pi]$	(ii) $\cos(\cos^{-1}x) = x$, $x \in [-1, 1]$
(iii) $\tan^{-1}\tan\theta = \theta$, for all $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$	(iii) $\tan(\tan^{-1}x) = x$, $x \in R$

2. (a) (i) $\sin^{-1}(-x) = -\sin^{-1}(x)$, $x \in [-1, 1]$
   (ii) $\cos^{-1}(-x) = \pi - \cos^{-1}(x)$, $x \in [-1, 1]$
   (iii) $\tan^{-1}(-x) = -\tan^{-1}x$, $x \in R$
3. (i) $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$, if $xy < 1$
   $= \pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right)$, if $xy > 1$, $x > 0$, $y > 0$
   $= \tan^{-1}\left(\frac{x+y}{1-xy}\right) - \pi$, if $xy > 1$, $x < 0$, $y < 0$
   (ii) $\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$, if $xy > -1$
   $= \tan^{-1}\left(\frac{x-y}{1+xy}\right) - \pi$, if $x < 0$, $y > 0$ & $xy < -1$
   $= \pi + \tan^{-1}\left(\frac{x-y}{1+xy}\right)$, if $x > 0$, $y < 0$, $xy < -1$

Note : If $x_1, x_2, x_3, \ldots\ldots x_n \in R$, then

$$\tan^{-1}x_1 + \tan^{-1}x_2 + \ldots\ldots\ldots\ldots + \tan^{-1}x_n = \tan^{-1}\left(\frac{S_1 - S_3 + S_5 - S_7 + \ldots\ldots\ldots}{1 - S_2 + S_4 - S_6 + \ldots\ldots\ldots}\right)$$

where S_k denotes the sum of products of $x_1, x_2, \ldots\ldots x_n$ taken k at a time.

Chap-04 - Complex Numbers & Quadratic Equations

Formulae Chart

1. Conjugate of a complex number $z = a + ib$ is denoted by $\bar{z}$ and is equal to $a - ib$.
   For any three complex numbers z, z_1 and z_2,
   (i) $z\bar{z} = (\text{Re}\,z)^2 + (\text{Im}\,z)^2$
   (ii) If z is purely real then $z = \bar{z}$
   (iii) If z is purely imaginary then $z = -\bar{z}$
   (iv) $\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$
   (v) $\overline{z_1 - z_2} = \bar{z}_1 - \bar{z}_2$
   (vi) $\overline{z_1 z_2} = \bar{z}_1\bar{z}_2$
   (vii) $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}$, $z_2 \neq 0$
2. The modulus of a complex number $z = a + ib$ is denoted by $|z|$ and is equal to $\sqrt{a^2+b^2}$
   For any three complex numbers z, z_1 and z_2,
   (i) $|z| = 0$ iff $z = 0$
   (ii) $z\bar{z} = |z|^2$ or $z^{-1} = \frac{\bar{z}}{|z|^2}$, $z \neq 0$
   (iii) $|z| = |\bar{z}| = |-z|$
   (iv) $-|z| \leq \text{Re}\,z \leq |z|$ and $-|z| \leq \text{Im}\,z \leq |z|$
   (v) $|z_1z_2| = |z_1|\,|z_2|$
   (vi) $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$, $z_2 \neq 0$
   (vii) $|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2(|z_1|^2 + |z_2|^2)$
   (viii) $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2 + 2\text{Re}(z_1\bar{z}_2)$
   (ix) $|z_1 - z_2|^2 = |z_1|^2 + |z_2|^2 - 2\text{Re}(z_1\bar{z}_2)$

3. If $z = a + ib$ is a complex number, then

$$\sqrt{a+ib} = \begin{cases} \pm\left(\sqrt{\frac{1}{2}\left(\sqrt{a^2+b^2}+a\right)} + i\sqrt{\frac{1}{2}\left(\sqrt{a^2+b^2}-a\right)}\right), & \text{if } b \text{ is positive} \\ \pm\left(\sqrt{\frac{1}{2}\left(\sqrt{a^2+b^2}+a\right)} - i\sqrt{\frac{1}{2}\left(\sqrt{a^2+b^2}-a\right)}\right), & \text{if } b \text{ is negative} \end{cases}$$

4. a. If $ax^2 + bx + c = 0$, $a \neq 0$ is a quadratic equation with real coefficients, then its roots are given by
   $\alpha, \beta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ or $\alpha, \beta = \frac{-b \pm \sqrt{D}}{2a}$
   where $D = b^2 - 4ac$ is the discriminant of the equation. If $D < 0$, then the roots are imaginary and are given by $\alpha = \frac{-b \pm \sqrt{4ac - b^2}\,i}{2a}$ (i.e., imaginary roots occur conjugate pairs)
   b. D = a perfect square, then roots of $ax^2 + bx + c = 0$ are rational only when the coefficients a, b, c are rationals otherwise roots are irrational.
   Irrational roots of an equation always occur in conjugate pairs, i.e., if $p + \sqrt{q}$ is one root of $ax^2 + bx + c = 0$, then other root must be $p - \sqrt{q}$.
   c. If roots of $ax^2 + bx + c = 0$ are equal in magnitude and opposite in signs, then sum of roots $= 0 = -\frac{b}{a}$
   $\Rightarrow$ b = coefficient of $x = 0$.
   d. To have both roots positive of $ax^2 + bx + c = 0$, $D > 0$, sum of roots > 0, product of roots > 0.
   e. To have both roots negative of $ax^2 + bx + c = 0$, $D > 0$, sum of roots < 0, product of roots > 0.
   f. $ax^2 + bx + c > 0$ for all $x \in R$, $a > 0$, $D < 0$.
   g. $ax^2 + bx + c < 0$ for all $x \in R$, $a < 0$, $D < 0$.

Chap-05 - Sequences and Series

Formulae Chart

1. For Arithmetic progressions :
   (i) $a_n = a + (n-1)d$
   (ii) $d = \frac{b-a}{n-1}$, if b is the n^{th} term
   (iii) $S_n = \frac{n}{2}[2a + (n-1)d]$
   $= \frac{n}{2}[a + l]$
   Where a = first term
   d = common difference
   n = number of terms
   l = last term
2. For Geometric progressions :
   (i) $a_n = ar^{n-1}$
   (ii) $r = \left(\frac{b}{a}\right)^{\frac{1}{n-1}}$, if b is the n^{th} term
   (iii) $S_n = \frac{a(1-r^n)}{1-r} = \frac{a(r^n - 1)}{r-1}$, $r \neq 1$
   (iv) $S_\infty = \frac{a}{1-r}$ provided $|r| < 1$ i.e. $-1 < r < 1$
   Where a = first term
   r = common ratio
   b = n^{th} term.
   n = number of terms
3. For two positive numbers a and b
   A.M. $\geq$ G.M.
   Sums in terms of special series :
   (i) Sum of first n natural numbers $= \sum_{k=1}^{n} k = \frac{n(n+1)}{2}$
   (ii) Sum of squares of the first n natural numbers $= \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$
   (iii) Sum of cubes of the first n natural numbers $= \sum_{k=1}^{n} k^3 = \left[\frac{n(n+1)}{2}\right]^2$

Chap-06 - Permutation and Combination

Formulae Chart

1. ${}^nP_r = \frac{n!}{(n-r)!}$, where $0 \leq r \leq n$
2. The number of permutations of n different things taken all at a time, where repetition is allowed is n^r.
3. The number of permutations of n distinct things, taken all at a time is $n!$.
4. The number of permutations of n objects taken all at a time, where k_1 are alike of one kind, k_2 are alike of second kind, k_3 are alike of third kind, ..., k_r are alike of r^{th} kind such that $k_1 + k_2 + k_3 + \ldots\ldots\ldots\ldots\ldots\ldots + k_r = n$ is $\frac{n!}{k_1!\,k_2!\,k_3!\ldots\ldots k_r!}$
5. ${}^nC_r = \frac{n!}{r!(n-r)!}$, where $0 \leq r \leq n$
6. ${}^nP_r = {}^nC_r\,r!$
7. ${}^nC_r = {}^nC_{n-r}$, where $0 \leq r \leq n$
8. If ${}^nC_x = {}^nC_y$, then $x = y$ or $x + y = n$
9. ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$, where $1 \leq r \leq n$
10. $\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}$, $r \geq 1$

Chap-07 - Binomial Theorem

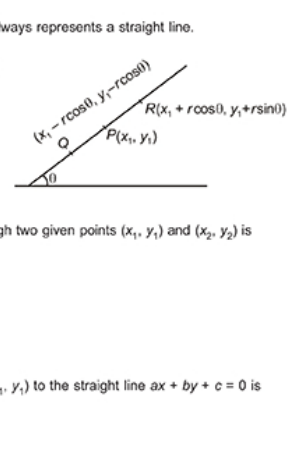
Formulae Chart

1. The general term (T_{r+1}) in the expansion of $(x + a)^n$ is given by $T_{r+1} = {}^nC_r\,x^{n-r}a^r$.
2. The middle term(s) in the expansion of $(x + a)^n$ is/are
   (i) $\frac{n}{2} + 1$ if n is even
   (ii) $\frac{n+1}{2}$, $\frac{n+3}{2}$ if n is odd.
3. Among the binomial coefficients, the greatest binomial coefficients is
   (i) ${}^nC_{\frac{n}{2}}$ if n is even
   (ii) ${}^nC_{\frac{n-1}{2}} = {}^nC_{\frac{n+1}{2}}$ if n is odd
4. ${}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots\ldots + {}^nC_n = 2^n$
5. (a) ${}^nC_0 + {}^nC_2 + {}^nC_4 + \ldots\ldots + {}^nC_{n-1} = 2^{n-1} = {}^nC_1 + {}^nC_3 + \ldots\ldots + {}^nC_n$, if n is odd.
   (b) ${}^nC_0 + {}^nC_2 + {}^nC_4 + \ldots\ldots + {}^nC_n = 2^{n-1} = {}^nC_1 + {}^nC_3 + \ldots\ldots + {}^nC_{n-1}$, if n is even.

Chap-08 - Straight Lines

Formulae Chart

1. **Straight line**
   (i) Every equation of first degree in x and y always represents a straight line.
   (ii) The equation of straight line passing through (x_1, y_1) and making an angle θ with positive direction of x-axis can be written parametrically as
   $\frac{x - x_1}{\cos\theta} = \frac{y - y_1}{\sin\theta} = r$
   Any point on this distant r from (x_1, y_1) is
   $(x_1 + r\cos\theta, y_1 + r\sin\theta)$



   (iii) The equation of straight line passing through two given points (x_1, y_1) and (x_2, y_2) is
   $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$
   i.e. $\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1}$
   (iv) The length of perpendicular drawn from (x_1, y_1) to the straight line $ax + by + c = 0$ is
   $\left|\frac{ax_1 + by_1 + c}{\sqrt{a^2+b^2}}\right|$
   Distance between two parallel lines $ax + by + c = 0$ and $ax + by + c_1 = 0$ is
   $\left|\frac{c - c_1}{\sqrt{a^2+b^2}}\right|$
   (v) The equation of straight line passing through the point of intersection of two given lines $L_1 = 0$ and $L_2 = 0$ may be written as
   $L_1 + kL_2 = 0$
   (vi) Relative position of the points (x_1, y_1) and (x_2, y_2) with respect to the line $ax + by + c = 0$; the points (x_1, y_1) and (x_2, y_2) lie on the same side or opposite side of the line $ax + by + c = 0$ according as $ax_1 + by_1 + c$ and $ax_2 + by_2 + c$ have the same sign or opposite sign.
   (vii) The equations of the bisector of the angles between the lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ ($a_1b_2 \neq a_2b_1$) are given by
   $\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2+b_1^2}} = \pm\frac{a_2x + b_2y + c_2}{\sqrt{a_2^2+b_2^2}}$

2. (i) The foot of the perpendicular (h, k) from (x_1, y_1) to the straight line $ax + by + c = 0$ is given by
   $\frac{h - x_1}{a} = \frac{k - y_1}{b} = -\frac{(ax_1 + by_1 + c)}{a^2 + b^2}$
   (ii) The image or reflection of the point (x_1, y_1) in the line $ax + by + c = 0$ is (h, k) where
   $\frac{h - x_1}{a} = \frac{k - y_1}{b} = \frac{-2(ax_1 + by_1 + c)}{a^2 + b^2}$

Chap-09 - Circle

Formulae Chart

1. **Equation of Circle**
   (i) The equation of the circle on the line segment joining (x_1, y_1) and (x_2, y_2) as diameter is $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$
   (ii) The equations of the circle with centre at (h, k) and touching axes and y-axis are
   $x^2 + y^2 - 2hx - 2ky + h^2 = 0$ and $x^2 + y^2 - 2hx - 2ky + k^2 = 0$ respectively.
   (iii) The equation of the circle touching both axes and radius 'a' is
   $x + y^2 \pm 2ax \pm 2ay + a^2 = 0$
   (iv) The equation of a circle passing through the point of intersection of a given circle $S = 0$ and a given line $L = 0$ is $S + \lambda L = 0$, where λ is a constant to be determined by the given condition.
   (v) The equation of a circle passing through the point of intersection of two given circles $S_1 = 0$ and $S_2 = 0$ is given by $S_1 + \lambda S_2 = 0$, where $\lambda \neq -1$ and leading coefficients of x^2 and y^2 in S_1 and S_2 are unity.
2. **Equations of Tangent and Normal**
   (i) The equation of the tangent and the normal to the circle $x^2 + y^2 = a^2$ at (x_1, y_1) is
   $xx_1 + yy_1 = a^2$ and $\frac{x}{x_1} - \frac{y}{y_1} = 0$ respectively.
   (ii) The equation of the tangent and the normal to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ at (x_1, y_1) is $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$
   and $\frac{x - x_1}{x_1 + g} = \frac{y - y_1}{y_1 + f}$ respectively.
   (iii) The straight line $y = mx + c$ will be tangent to the circle $x^2 + y^2 = a^2$ if $c^2 = a^2(1 + m^2)$.
   Then the equation of the tangent to the circle $x^2 + y^2 = a^2$ in slope form may be written as
   $y = mx \pm a\sqrt{1 + m^2}$ and the points of contact are $\left(\mp\frac{am}{\sqrt{1+m^2}}, \pm\frac{a}{\sqrt{1+m^2}}\right)$
3. **Parametric Equation**
   (i) Parametric equation of circle $x^2 + y^2 = a^2$ is $x = a\cos\theta$, $y = a\sin\theta$.
   (ii) Parametric equation of circle $(x - x_1)^2 + (y - y_1)^2 = a^2$ is $x = x_1 + a\cos\theta$, $y = y_1 + a\sin\theta$.

Chap-10 - Conic Section

Formulae Chart

1. **Parabola**
   (i) Parametric equation of $y^2 = 4ax$ is $x = at^2$, $y = 2at$.
   (ii) Parametric equation of $x^2 = 4ay$ is $x = 2at$, $y = at^2$.
   (iii) Equation of tangent having slope m to $y^2 = 4ax$ is $y = mx + \frac{a}{m}$
   (iv) Equation of tangent having slope m to $x^2 = 4ay$ is $y = mx - am^2$.
   (v) Equation of tangent at $(at^2, 2at)$ is $y = \frac{x}{t} + at$.
   (vi) Equation of normal having slope m is $y = mx - 2am - am^3$.
   (vii) Equation of normal at $(at^2, 2at)$ is $y = -tx + 2at + at^3$.
   (viii) $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$ are end points of focal chord if $t_1t_2 = -1$.
2. **Ellipse**
   (i) Parametric equation of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $x = a\cos\theta$, $y = b\sin\theta$
   (ii) Equation of tangent having slope m to $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $y = mx \pm \sqrt{a^2m^2 + b^2}$
   (iii) Equation of tangent at (x_1, y_1) is $\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$.
   (iv) Equation of tangent at $(a\cos\theta, b\sin\theta)$ is $\frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1$.
   (v) Equation of normal at (x_1, y_1) is $\frac{x - x_1}{\frac{x_1}{a^2}} = \frac{y - y_1}{\frac{y_1}{b^2}}$
   (vi) Equation of normal at $(a\cos\theta, b\sin\theta)$ is $ax\sec\theta - by\text{cosec}\,\theta = a^2 - b^2$.
3. **Hyperbola**
   (i) Parametric equation of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $x = a\sec\theta$, $y = b\tan\theta$.
   (ii) Equation of tangent having slope m to $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $y = mx \pm \sqrt{a^2m^2 - b^2}$
   (iii) Equation of tangent at (x_1, y_1) is $\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$.

   (iv) Equation of tangent at $(a\sec\theta, b\tan\theta)$ is $\frac{x\sec\theta}{a} - \frac{y\tan\theta}{b} = 1$.
   (v) Equation of normal at (x_1, y_1) is $\frac{x - x_1}{\frac{x_1}{a^2}} = \frac{y - y_1}{-\frac{y_1}{b^2}}$
   (vi) Equation of normal at $(a\sec\theta, b\tan\theta)$ is $ax\cos\theta + by\cot\theta = a^2 + b^2$.

Chap-11 Statistics

Formulae Chart

- Mean deviation for ungrouped data
  (a) about mean
  M.D. $(\bar{x}) = \frac{\sum|x_i - \bar{x}|}{n}$
  (b) about median
  M.D. $(M) = \frac{\sum|x_i - M|}{n}$
- Mean deviation for grouped data
  (a) about mean
  M.D. $(\bar{x}) = \frac{\sum f_i|x_i - \bar{x}|}{N}$
  (b) about median
  M.D. $(M) = \frac{\sum f_i|x_i - M|}{N}$, where $N = \sum f_i$
- Variance and standard deviation for ungrouped data
  $\sigma^2 = \frac{1}{n}\sum(x_i - \bar{x})^2$
  $\sigma = \sqrt{\frac{1}{n}\sum(x_i - \bar{x})^2}$
- Variance and standard deviation of a discrete frequency distribution.
  $\sigma^2 = \frac{1}{N}\sum f_i(x_i - \bar{x})^2$, $\sigma = \sqrt{\frac{1}{N}\sum f_i(x_i - \bar{x})^2}$
- Variance and standard deviation of a continuous frequency distribution:
  $\sigma^2 = \frac{1}{N}\sum f_i(x_i - \bar{x})^2$, $\sigma = \frac{1}{N}\sqrt{N\sum f_ix_i^2 - \left(\sum f_ix_i\right)^2}$
- Shortcut to find variance and standard deviation.
  $\sigma^2 = \frac{h^2}{N^2}\left[N\sum f_iy_i^2 - \left(\sum f_iy_i\right)^2\right]$
  $\sigma = \frac{h}{N}\sqrt{N\sum f_iy_i^2 - \left(\sum f_iy_i\right)^2}$, where $y_i = \frac{x_i - A}{h}$
- Co-efficient of variation $(\text{C.V.}) = \frac{\sigma}{\bar{x}} \times 100$, $\bar{x} \neq 0$.

Chap-12 - Matrices and Determinants

Formulae Chart

1. (i) **Trace of a square matrix**
   $\text{tr}A = \sum_{i=1}^{n} a_{ii} = a_{11} + a_{22} + \ldots\ldots + a_{nn}$
   where $A = [a_{ij}]_{n \times n}$
   (ii) $\text{tr}(\lambda A) = \lambda\,\text{tr}A$
   (iii) $\text{tr}(A + B) = \text{tr}A + \text{tr}B$
   (iv) $\text{tr}(AB) = \text{tr}(BA)$
2. **Transpose of matrix (A)**
   (i) $(A^T)^T = A$
   (ii) $(A + B)^T = A^T + B^T$, A and B being of same size
   (iii) $(KA)^T = KA^T$, K be any complex number
   (iv) $(AB)^T = B^TA^T$, A and B being conformable for multiplication
3. **Conjugate of a matrix**
   (i) If $A = [a_{ij}]_{m \times n}$ then $\bar{A} = [\bar{a}_{ij}]_{m \times n}$
   (ii) $\overline{(\bar{A})} = A$
   (iii) $\overline{(A + B)} = \bar{A} + \bar{B}$
   (iv) $\overline{(KA)} = \bar{K}\bar{A}$, K being any complex number
   (v) $\overline{AB} = \bar{A}\bar{B}$, A and B being conformable for multiplication.
4. (i) If $A^T = A$, then A is symmetric matrix $[a_{ij} = a_{ji}]$
   (ii) If $A^T = -A$, then A is skew-symmetric matrix $[a_{ij} = -a_{ji}]$
   (iii) Every square matrix can be uniquely expressed as the sum of symmetric and skew-symmetric matrix.
   (i.e. $A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$
   $\frac{1}{2}(A + A^T)$ = Symmetric matrix
   $\frac{1}{2}(A - A^T)$ = Skew symmetric matrix
5. **Orthogonal matrix**
   If $AA^T = I$ where I is identity matrix where $A \to$ orthogonal matrix

6. **Adjoint of a matrix**
   $A = [a_{ij}]_{n \times n}$, a square matrix
   then $B = [C_{ij}]_{n \times n}$, where A_{ij} = cofactor of element a_{ij} in the determinant $|A|$
   then $\text{Adj}A = B^T$
   (i) $A(\text{adj}A) = (\text{adj}A)A = |A|I_n$, $I_n \to n$ rowed unit matrix
   (ii) $\text{adj}(O) = O$ and $\text{adj}I_n = I_n$, O being a zero matrix of order n
   (iii) $(\text{adj}A)^T = \text{adj}(A^T)$, $(\text{adj}A)^{-1} = \text{Adj}(A^{-1})$
   (iv) $\text{adj}(AB) = (\text{adj}B)\,(\text{adj}A)$
   (v) $\det(\text{adj}A) = (\det A)^{n-1}$, n being the order of matrix A
7. **Inverse of a matrix**
   If A be n-row square matrix, such that $AB = BA = I$, then B is called the inverse of A, and A is said to be invertible.
   (i) $A^{-1} = \frac{1}{\det A}\text{adj}A$, $|A| \neq 0$
   If $|A| = 0$, matrix is called singular
   If $|A| \neq 0$, matrix is called non-singular
   (ii) $(AB)^{-1} = B^{-1}A^{-1}$
   (iii) $(A^{-1})^T = (A^T)^{-1}$
   (iv) $(A^{-1})^{-1} = (A^{-1})^{-1}$
   (v) $\det(A^{-1}) = (\det A)^{-1}$

DETERMINANTS

1. If a determinant is of n order then it contains n rows and n columns.
   Number of constituents in determinant $= n^2$
2. **Cramer's Rule :**
   (a) For Non-Homogeneous system of equation
   Let us consider a system of equations
   $a_1x + b_1y + c_1z = d_1$
   $a_2x + b_2y + c_2z = d_2$
   $a_3x + b_3y + c_3z = d_3$

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix},\ \Delta_1 = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix},\ \Delta_2 = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix},\ \Delta_3 = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

   $x = \frac{\Delta_1}{\Delta}$, $y = \frac{\Delta_2}{\Delta}$, $z = \frac{\Delta_3}{\Delta}$
   (i) If $\Delta \neq 0$, the system will have unique finite solution, so equation is consistent
   (ii) If $\Delta = 0$, at least any of $\Delta_1, \Delta_2, \Delta_3$ be non zero. The system has no solution i.e., equations are inconsistent.
   (iii) If $\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$, the system will have infinite solution and at least one co-factor of Δ is non-zero. i.e., equations are consistent.
   (b) For Homogeneous system of equation
   $a_1x + b_1y + c_1z = 0$
   $a_2x + b_2y + c_2z = 0$
   $a_3x + b_3y + c_3z = 0$
   then $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$, $\Delta_1 = \Delta_2 = \Delta_3 = 0$
   If $\Delta \neq 0$, system will have unique solution called trivial solution.
   If $\Delta = 0$, system will have infinite solution called non-trivial solution.

Chap-13- Limit, Continuity and Derivatives

Formulae Chart

1. **Properties of limits (these properties are used when either both limits are finite or both are indeterminate)**
   (i) $\lim_{x \to a}[f(x) \pm g(x)] = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)$
   (ii) $\lim_{x \to a}[f(x).g(x)] = \lim_{x \to a} f(x).\lim_{x \to a} g(x)$
   (iii) $\lim_{x \to a}\frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$, $\lim_{x \to a} g(x) \neq 0$
   (iv) $\lim_{x \to a}[k f(x)] = k\lim_{x \to a} f(x)$
   (v) $\lim_{x \to a}|f(x)| = \left|\lim_{x \to a} f(x)\right|$
   (vi) If $f(x) \leq g(x)$, $\forall x$
   $\Rightarrow \lim_{x \to a} f(x) \leq \lim_{x \to a} g(x)$
2. **Some standard results :**
   (i) $\lim_{x \to 0}\frac{\sin x}{x} = 1$, ($x$ in radians)
   (ii) $\lim_{x \to a}\frac{x^n - a^n}{x - a} = na^{n-1}$, $n \in Q$
   (iii) $\lim_{x \to 0}\frac{\tan x}{x} = 1$
   (iv) $\lim_{x \to 0}\frac{e^x - 1}{x} = 1$
   (v) $\lim_{x \to 0}\frac{a^x - 1}{x} = \ell n\,a$, $a > 0$, $a \neq 1$
   (vi) $\lim_{x \to 0}(1 + x)^{\frac{1}{x}} = e$
   (vii) $\lim_{x \to a}[1 + f(x)]^{g(x)} = e^{\lim_{x \to a} f(x).g(x)}$
   If $f(x) \to 0$ and $g(x) \to \infty$ as $x \to a$
3. **De' L'Hospital rule :**
   If $\lim_{x \to a}\frac{f(x)}{g(x)}$ is $\frac{0}{0}$ or $\frac{\infty}{\infty}$ form then L' Hospital rule is applicable. We differentiate numerator and denominator separately to evaluate the limit.

Continuity

(a) Let $f(x)$ be a real-valued function and $a \in R$; then function $f(x)$ is said to be continuous at $x = a$ if
   (i) $f(a)$ is defined
   (ii) $\lim_{x \to a} f(x)$ exists

   (iii) $\lim_{x \to a} f(x) = f(a)$
   Note :
   (i) Also, we can say that, a function $f(x)$ is said to be continuous at $x = a$ if $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$.
   (ii) If any one of the above three conditions is not satisfied then $f(x)$ is discontinuous at $x = a$.
   (iii) Geometrically, if we draw the graph of a function without raising pen or pencil, then we say that it is a continuous function.
(b) Continuity on an interval
   (i) Open interval : A function $f(x)$ is said to be continuous on (a, b) if it is continuous at each point of (a, b).
   (ii) Closed interval : A function $f(x)$ is said to be continuous on $[a, b]$ if
   (a) It is continuous on (a, b)
   (b) $\lim_{x \to a^+} f(x) = f(a)$
   (c) $\lim_{x \to b^-} f(x) = f(b)$
4. **Continuity of composite function :** If $f(x)$ is continuous at $x = a$ and $g(x)$ is continuous at $x = f(a)$, then $g(f(x))$ is continuous at $x = a$.
   Let $f(x) = \sin x$ and $g(x) = |x|$ are continuous at $x = 0$
   Thus the function $g(f(x)) = |\sin x|$ will also be continuous at $x = 0$
5. **Intermediate value theorem**
   Let $f(x)$ be a continuous real valued function and a, $b \in R$.
   (i) If $f(a)$ and $f(b)$ are of opposite signs, then there exists at least one solution of the equation $f(x) = 0$ in (a, b).
   (ii) If k is any real number between $f(a)$ and $f(b)$ then there exists at least one solution of the equation $f(x) = k$ in (a, b).
6. **Differentiability**
   Let $f(x)$ be a real valued function and $a \in R$. A function $f(x)$ is said to be differentiable at $x = a$ if $f'(a)$ exists
   i.e. $\lim_{h \to 0}\frac{f(a+h) - f(a)}{h} = \lim_{h \to 0}\frac{f(a-h) - f(a)}{-h}$
   i.e. L.H.D. = R.H.D.
   Thus, $f'(a) = \lim_{h \to 0}\frac{f(a+h) - f(a)}{h}$
7. **Differentiability on an interval :** Let $f(x)$ be a real valued function and a, $b \in R$
   (i) Open interval : A function $f(x)$ is said to be differentiable on (a, b) if it is differentiable at each point of (a, b).
   (ii) Closed interval : A function $f(x)$ is said to be a differentiable on $[a, b]$ if

   (a) It is differentiable on (a, b)
   (b) $\lim_{h \to 0}\frac{f(a+h) - f(a)}{h}$ is finite
   (c) $\lim_{h \to 0}\frac{f(b-h) - f(b)}{-h}$ is finite
   (iii) $\lim_{x \to a}\frac{f(x) - f(a)}{x - a}$ is finite
   Note :
   1. Geometrically, a function $f(x)$ is said to be differentiable at $x = a$ if there exists a unique tangent of finite slope at $x = a$
   2. A function $f(x)$ is not differentiable at $x = a$ if the there is a corner point at $x = a$.

Chap-14 - Applications of Derivatives

Formulae Chart

1. The equation of the tangent to a curve at (x_1, y_1)
   $y - y_1 = m(x - x_1)$
   The equation of the normal to a curve at (x_1, y_1)
   $y - y_1 = -\frac{1}{m}(x - x_1)$
   where $m = \left.\frac{dy}{dx}\right|_{(x_1, y_1)}$
2. The angle of intersection of two curves
   $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right|$
   where m_1, m_2 are the slops of the tangents to the curves at the point of intersection.
3. Increasing and decreasing functions :
   - If $f(x_1) < f(x_2)$, $\forall x_1 < x_2$, $x_1, x_2 \in D$, then $f(x)$ is strictly increasing.
   - If $f'(x) > 0$, $\forall x \in D$, then $f(x)$ is strictly increasing function.
   - If $f(x_1) \leq f(x_2)$, $\forall x_1 < x_2$, $x_1, x_2 \in D$, then $f(x)$ is increasing function.
   - If $f'(x) \geq 0$, $\forall x \in D$, then $f(x)$ is increasing function.
   - If $f(x_1) > f(x_2)$, $\forall x_1 < x_2$, $x_1, x_2 \in D$, then $f(x)$ is strictly decreasing function.
   - If $f'(x) < 0$, $\forall x \in D$, then $f(x)$ is strictly decreasing function.
   - If $f(x_1) \geq f(x_2)$, $\forall x_1 < x_2$, $x_1, x_2 \in D$, then $f(x)$ is decreasing function.
   - If $f'(x) \leq 0$, $\forall x \in D$, then $f(x)$ is decreasing function.
4. **Maxima and Minima**
   - If $f(x_0 + h) > f(x_0)$ and $f(x_0 - h) > f(x_0)$, $h \to 0^+$, then $f(x)$ has relative minima at $x = x_0$
   - If $f(x_0 + h) < f(x_0)$ and $f(x_0 - h) < f(x_0)$, $h \to 0^+$, then $f(x)$ has relative maxima at $x = x_0$
   - If $f(x)$ is differentiable function, then for maxima or minima $f'(x) = 0 \Rightarrow x = x_1, x_2, \ldots\ldots x_n$
   - **First derivative test :** If f' changes its sign from positive to negative about a critical point then it has point of local maxima.
   - If f' changes sign from negative to positive then it has a point of minima.
   - If there is no sign change about the critical point then it has neither maxima nor minima at that point.
   - n^{th} derivative test : Solve $f'(x) = 0 \Rightarrow x = x_1, x_2, \ldots\ldots x_n$
   - If $f''(x_1) > 0$, then $f(x)$ has relative minima at $x = x_1$
   - If $f''(x_1) < 0$, then $f(x)$ has relative maxima at $x = x_1$
   - If $f''(x_1) = 0$, find $f'''(x)$

   - If $f'''(x_1) \neq 0 \Rightarrow$ neither maxima nor minima it is called point of inflexion.
   - If $f'''(x_1) = 0$, find $f^{iv}(x)$
   Repeat the process till we get $f^{(n)}(x_1) \neq 0$.
   If n is odd $\Rightarrow$ neither maxima nor minima
   If n is even $\Rightarrow$ $f^{(n)}(x_1) > 0 \Rightarrow$ minima at $x = x_1$
   $f^{(n)}(x_1) < 0 \Rightarrow$ maxima at $x = x_1$
5. **Rolle's theorem :**
   Let f be a real valued function which is
   (i) Continuous on bounded and closed interval $[a, b]$
   (ii) Differentiable at every point x in the open interval (a, b)
   (iii) $f(a) = f(b)$
   then there exists at least one point c in (a, b)
   such that $f'(c) = 0$
6. **Lagrange's Mean Value Theorem :** Let f be a real valued function which is
   (i) Continuous on $[a, b]$
   (ii) Differentiable in (a, b)
   Then there exists at least one value of c, say c, such that
   $f'(c) = \frac{f(b) - f(a)}{b - a}$, $c \in (a, b)$

Chap-15 - Indefinite Integrals

Formulae Chart

1. **Integration by substitution :**
   Basic rule of substitution :
   $\int f(x)dx = \int f(\phi(z))\phi'(z)dz$
   Substitute $x = \phi(z)$
2. **Integration by parts :**
   If u and v be two functions of x then $\int uvdx = u\int vdx - \int\left(\frac{du}{dx}\int vdx\right)dx$ is known as formula for integration by parts.
   Note : In finding integrals by this method proper choice of u and v is essential. In general the function as u is taken which comes first in the word ILATE.
   where I - Inverse circular function
   L - Logarithmic function
   A - Algebraic function
   T - Trigonometric function
   E - Exponential function
   Note :
   - If both the functions are trigonometric, take that function as v whose integral is simpler.
   - If both the functions are algebraic take that function as 'u' whose $d.c.$ is simpler.
3. **Integration of rational function :**
   If integrand is of the form $\frac{P(x)}{Q(x)}$, where $P(x)$ and $Q(x)$ are polynomials of x.
   Case I. If $\frac{P(x)}{Q(x)}$ is proper rational function (i.e. highest power of x in $Q(x) > P(x)$)
   Integration by partial fraction is done.
   Case II. If $\frac{P(x)}{Q(x)}$ is improper rational function (i.e. highest power of x in $Q(x) < P(x)$)
   Then it is make proper rational function by simple division and then partial fraction is used
   i.e., $\frac{P(x)}{Q(x)} = \frac{f(x) \cdot Q(x) + R(x)}{Q(x)} = f(x) + \frac{R(x)}{Q(x)}$
   $f(x)$ is integrated by general rule and $\frac{R(x)}{Q(x)}$ is integrated by partial fraction.
4. **Integration of certain irrational expressions :**
   (i) If the integrand is a rational function of fractional powers of an independent variable x i.e.

   the function $R\left(x, x^{\frac{m}{n}}, \ldots\ldots, x^{\frac{r}{s}}\right)$. Substitute $x = t^k$, m is L.C.M. of $q_1, q_2, \ldots q_n$.
   (ii) If the integrand is a rational function of x and fractional powers of a linear fractional function of the form $\frac{ax+b}{cx+d}$ i.e. $R\left(x, \left(\frac{ax+b}{cx+d}\right)^{\frac{m_1}{n_1}}, \left(\frac{ax+b}{cx+d}\right)^{\frac{m_2}{n_2}}, \ldots\ldots, \left(\frac{ax+b}{cx+d}\right)^{\frac{m_k}{n_k}}\right)$
   substitute $\frac{ax+b}{cx+d} = t^m$ where m is L.C.M. of $(l_1, l_2, \ldots\ldots l_k)$.
5. **Integration of the form** $\int\frac{dx}{\sqrt{ax^2 + bx + c}}$
   (i) Integrals of the form $I = \int\frac{Mx + N}{\sqrt{ax^2 + bx + c}}dx$
   Substitute $x + \frac{b}{2a} = t$ and reduce it to the form $I = M_1\int\frac{tdt}{\sqrt{at^2 + k}} + N_1\int\frac{dt}{\sqrt{at^2 + k}}$, M_1, N_1 are new constants.
   (ii) Integrals of the form $I = \int\frac{P_n(x)}{\sqrt{ax^2 + bx + c}}dx$
   $P_n(x)$ is polynomial of degree m
   $I = P_{n-1}(x)\sqrt{ax^2 + bx + c} + \lambda\int\frac{dx}{\sqrt{ax^2 + bx + c}}$
   where $P_{n-1}(x)$ is polynomial of degree $(n - 1)$ and λ is constant.
   Coefficient of $P_{n-1}(x)$ is determined by the method of undetermined coefficients.
   (iii) Integrals of the form $\int\frac{dx}{(x - \alpha)^n\sqrt{ax^2 + bx + c}}$
   Substitute $(x - \alpha) = \frac{1}{t}$
   Note : For the integration of the form $\int\sqrt{ax^2 + bx + c}\,dx$ or $\int\frac{dx}{\sqrt{ax^2 + bx + c}}$ or $\int\frac{dx}{ax^2 + bx + c}$
   1st convert $ax^2 + bx + c$ in the form $A^2 + X^2$, $X^2 - A^2$ or $A^2 - X^2$, where A is constant and X is $px + q$.
   Following results is used :
   (i) $\int\frac{dx}{\sqrt{x^2 + a^2}} = \log\left|x + \sqrt{x^2 + a^2}\right| + C$
   (ii) $\int\sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + C$
   (iii) $\int\sqrt{x^2 + a^2}\,dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2}\log\left|x + \sqrt{x^2 + a^2}\right| + C$
   (iv) $\int\frac{1}{x^2 - a^2}\,dx = \frac{1}{2a}\log\left|\frac{x - a}{x + a}\right| + C$

6. **Integration of the form of** $\int x^m(a + bx^n)^p dx$, m, n, p are rational numbers.
   Case 1. If p is positive integer then integrand is expanded by binomial expansion.
   Case 2. If P is negative integer substitute $x = t^k$ where k is L.C.M. of denominator of m and n.
   Case 3. $\frac{m+1}{n}$ is an integer. Put $a + bx^n = t^\alpha$ where α is denominator of the fraction P.
   Case 4. $\frac{m+1}{n} + P$ is an integer, put $a + bx^n = t^\alpha x^n$ where α is denominator of fraction P.
7. **Integration of trigonometric function.**
   (i) Integration of the form $\int\sin^m x \cdot \cos^n x\,dx$
   (a) m = odd positive integer, substitute $z = \cos x$.
   (b) n = odd positive integer, substitute $z = \sin x$.
   (c) If both m and n are odd positive integer put $z = \sin x$ or $\cos x$
   (d) If m and n both are not odd positive integer
   (1) If $m + n$ is even negative integer put $z = \tan x$.
   (2) If $m + n$ is even positive integer, express the integrands as the algebraic sum of sines and cosines of multiple angles using formula
   $\sin^2 x = \frac{1 - \cos 2x}{2}$, $\cos^2 x = \frac{1 + \cos 2x}{2}$, $\sin^3 x = \frac{3\sin x - \sin 3x}{4}$, $\cos^3 x = \frac{3\cos x + \cos 3x}{4}$
   (ii) Integral of the form $\int\sec^m x\tan^n x\,dx$ or $\int\text{cosec}^m x\cot^n x\,dx$
   (a) If m is even positive integer, put $z = \tan x$ in I_1 or $z = \cot x$ in I_2
   (b) If n is even positive integer then put $\sec^2 x - 1$ in place of $\tan^2 x$ in I_1 and $\text{cosec}^2 x - 1$ in place of $\cot^2 x$ in I_2 and substitute $z = \tan x\,(I_1)$
   $= \cot x\,(I_2)$
   (c) $m = 0$, n is odd positive integer greater than 1 then method of integration by parts is used
   Ex. $\int\text{cosec}^3 x\,dx$, $\int\sec^3 x\,dx$
   (iii) If integrand contains any expression of the form $\sin x$, $\cos x$, $\tan x$, $\cot x$, $\sec x$, $\text{cosec}\,x$, where it is function of x. Ex. $\int\sin\left(\sqrt{x}\right)dx$
   (iv) Integration of the form $\int\frac{a\sin x}{a\sin x + b\cos x}dx$, $\int\frac{\cos x}{a\sin x + b\cos x}dx$ or $\int\frac{p\sin x + q\cos x}{a\sin x + b\cos x}dx$
   where $a \neq 0$, $b \neq 0$. Put numerator $= A$ (Denominator) $+ B$(D.C. of denominator)
   (v) Integration of the form $\int\frac{dx}{a\cos^2 x + 2h\sin x \cdot \cos x + b\sin^2 x}$ or $\int\frac{dx}{a + b\sin^2 x}$ or $\int\frac{dx}{a + b\cos^2 x}$. Divide numerator and denominator by $\cos^2 x$ and put $\tan x = z$.

   (vi) Integration of the form
   (i) $\int\frac{dx}{a\cos x + b\sin x + c}$ (ii) $\int\frac{dx}{a + b\cos x}$ (iii) $\int\frac{dx}{a + b\sin x}$
   Write $\sin x = \frac{2\tan\frac{x}{2}}{1 + \tan^2\frac{x}{2}}$, $\cos x = \frac{1 - \tan^2\frac{x}{2}}{1 + \tan^2\frac{x}{2}}$ and substitute $z = \tan\frac{x}{2}$.
   (vii) Trigonometric substitution, when integrand involves expression of the form
   (a) $\sqrt{a^2 - x^2}$, substitute $x = a\cos\theta$ or $a\sin\theta$.
   (b) $a^2 + x^2$ or $\sqrt{a^2 + x^2}$, substitute $x = a\tan\theta$ or $a\cot\theta$.
   (c) $\sqrt{x^2 - a^2}$, $x = a\sec\theta$ or $a\,\text{cosec}\,\theta$
   (d) $\sqrt{\frac{a - x}{a + x}}$ or $\sqrt{\frac{a + x}{a - x}}$, $x = a\cos 2\theta$.
   (e) $\sqrt{\frac{x}{a - x}}$ or $\sqrt{\frac{a - x}{x}}$, $x = a\sin^2\theta$ or $a\cos^2\theta$
   (f) $\sqrt{\frac{x}{a + x}}$ or $\sqrt{\frac{a + x}{x}}$, $x = a\tan^2\theta$ or $a\cot^2\theta$
   (g) $\sqrt{\frac{x - \alpha}{\beta - x}}$ or $\sqrt{(x - \alpha)(\beta - x)}$, substitute $x = \alpha\cos^2\theta + \beta\sin^2\theta$
8. **Integration of the form** $\int e^x[f(x) + f'(x)]dx$
   $\int e^x[f(x) + f'(x)]dx = e^x f(x) + c$
9. **Integrals of the form**
   (a) $\int\frac{px + q}{ax^2 + bx + c}dx$ (b) $\int\frac{px + q}{\sqrt{ax^2 + bx + c}}dx$
   (c) $\int(px + q)\sqrt{ax^2 + bx + c}\,dx$ (d) $\int\frac{px^2 + qx + r}{ax^2 + bx + c}dx$
   for (a), (b), (c), write $px + q = A\frac{d}{dx}(ax^2 + bx + c) + B$
   By equating similar power of x, find A and B.
   for (d) $px^2 + qx + r = A(ax^2 + bx + c) + B(2ax + b) + C$ find A, B, C and solve.
10. **Integrals of the form** $\int\frac{dx}{X\sqrt{Y}}$
    Use substitution as given in table

X	Y	Substitution
Linear	Linear	$Z^2 = Y$
Quadratic	Linear	$Z^2 = Y$
Linear	Quadratic	$Z = \frac{1}{X}$
Quadratic Quadratic	$Z = \frac{1}{x}$	or $Z = \frac{Y}{X}$

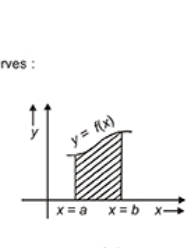
Chap-16 - Definite Integrals and their Applications

Formulae Chart

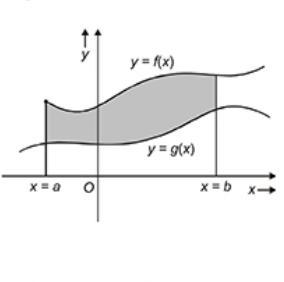
1. If $F(x)$ is the antiderivative of $f(x)$ then
   $\int_a^b f(x)dx = [F(x)]_a^b$
   $= F(b) - F(a)$
   this is called fundamental theorem of integral calculus.
2. Properties of definite integral :
   (i) $\int_a^b f(x)dx = \int_a^b f(t)dt$
   (ii) $\int_a^b f(x)dx = -\int_b^a f(x)dx$
   (iii) $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$, $a < c < b$ (this property is also true when c doesn't lie between a & b)
   (iv) $\int_0^a f(x)dx = \int_0^a f(a - x)dx$
   (v) $\int_a^b f(x)dx = \int_a^b f(a + b - x)dx$
   (vi) $\int_{-a}^{a} f(x)dx = \begin{cases} 0, & \text{if } f(x) \text{ is an odd function} \\ 2\int_0^a f(x)dx, & \text{if } f(x) \text{ is an even function} \end{cases}$
   (vii) $\int_0^{2a} f(x)dx = \int_0^a [f(x) + f(2a - x)]dx$
   $= \begin{cases} 2\int_0^a f(x)dx, & \text{if } f(2a - x) = f(x) \\ 0, & \text{if } f(2a - x) = -f(x) \end{cases}$
   (viii) $\frac{d}{dx}\int_{\phi(x)}^{\psi(x)} f(t)dt = f[\psi(x)]\psi'(x) - f[\phi(x)]\phi'(x)$ this is called Newton - Leibnitz rule.
   (ix) $\int_a^b f(x)dx = \int_a^b f(a + b - x)dx$
   (x) $\int_0^1 f(x)dx = \frac{1}{n}\int_0^n f\left(\frac{x}{n}\right)dx$

   (xi) If f is periodic function in properties x to a with period T, then
   $\int_0^{nT} f(x)dx = n\int_0^T f(x)dx$, $n \in N$
   (xii) $\int_a^{a+T} f(x)dx$ is independent of 'a'
   we can write
   $\int_a^{a+T} f(x)dx = \int_0^T f(x)dx$
   (xiii) $\int_{mT}^{nT} f(x)dx = (n - m)\int_0^T f(x)dx$, m, $n \in N$
   (xiv) $\int_{a+nT}^{b+nT} f(x)dx = \int_a^b f(x)dx$, $m \in N$
   (xv) (a) $\int_{a+nT}^{b+nT} f(x)dx = \int_a^b f(x)dx$ $\int_{mT}^{b+nT} f(x)dx = (n - m)\int_0^T f(x)dx$, m, $n \in N$
   (b) $\int_{a+nT}^{b+nT} f(x)dx = \int_0^T f(x + T)dx$, $n \in N$
   (xvi) $\int_{a+nT}^{b+nT} f(x)dx = (b - a)\int_0^1 [b(b - a)x + a]dx$
   (xvii) $\frac{d}{dx}\int_{\phi(x)}^{\psi(x)} f(t)dt = f[\psi(x)]\psi'(x) - f[\phi(x)]\phi'(x)$
   (xiv) $\int_a^b f(x)dx = (b - a)\int_0^1 f[(b - a)x + a]dx$
   (xv) $\frac{d}{dx}\int_{\phi(x)}^{\psi(x)} f(x, t)dt$ where $\frac{d}{dx}\int_{\phi(x)}^{\psi(x)}[f(x, t)]$ is derivative of f w.r.t. 'x' keeping 't' as constant.
   (xvi) $\frac{d}{dx}\int_{\phi(x)}^{\psi(x)} f(x, t)dt = f[x, \psi(x)]\psi'(x) - f[x, \phi(x)]\phi'(x) + \int_{\phi(x)}^{\psi(x)}\frac{\partial}{\partial x}f(x, t)dt$
3. Estimation of integral :
   (i) $\left|\int_a^b f(x)dx\right| \leq \int_a^b |f(x)|dx$
   (ii) If $g(x) < f(x) < h(x)$ is $[a, b]$, then $\int_a^b g(x)dx < \int_a^b f(x)dx < \int_a^b h(x)dx$
   (iii) (a) If f is an increasing function and has concave up graph in $[a, b]$ then
   $(b - a)f(a) < \int_a^b f(x)dx < (b - a)\frac{f(b) + f(a)}{2}$
   (b) If f is an increasing function and has concave down (or convex up) graph is $[a, b]$, then

   (iv) If M and m be the greatest and least values of function f on $[a, b]$, then,
   $m(b - a) \leq \int_a^b f(x)dx \leq M(b - a)$
   (v) If 'f' is a continuous function is $[a, b]$, then it assumes its mean value is $[a, b]$, that is
   $\frac{1}{b - a}\int_a^b f(x)dx = f(c)$, for same c, such that $a \leq c \leq b$
   This is known as mean value theorem of integral calculus.
   (vi) $\int_a^b f(x)g(x)dx \leq \sqrt{\int_a^b f^2(x)dx}\sqrt{\int_a^b g^2(x)dx}$
   This is known as Schwarz - Bunyakovsky inequality.
4. Definite integral as the limit of a sum :
   If $f(x)$ is continuous function on $[a, b]$, then
   $\int_a^b f(x)dx = \lim_{h \to 0} h\sum_{r=1}^{n} f(a + rh)$ where $nh = b - a$, $n \to \infty$ as $h \to 0$
   $\int_0^1 f(x)dx = \lim_{n \to \infty}\frac{1}{n}\sum_{r=1}^{n} f\left(\frac{r}{n}\right)$
5. Area under the curve or curves :
   (i) $A = \int_a^b f(x)dx$



   (ii) $A = \int_a^b [f(x) - g(x)]dx$



Chap-17_Differential Equation

Formulae Chart

1. A differential equation of type $\frac{dy}{dx} = F(x, y)$ is homogeneous differential equation if $F(\lambda x, \lambda y) = \lambda^0 F(x, y)$.
2. To solve homogeneous differential equation we put
   $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$
   or $x = vy \Rightarrow \frac{dx}{dy} = v + y\frac{dv}{dy}$
3. $\frac{dy}{dx} + Py = Q$ where P, Q are functions of x alone or constants is linear differential equation.
   I.F. $= e^{\int Pdx}$
   And the solution is given by
   $y.(\text{I.F.}) = \int Q(\text{I.F.})dx + c$
4. $\frac{dx}{dy} + Px = Q$ where P, Q are the functions of y only or constants
   I.F. $= e^{\int Pdy}$
   And solution is $x(\text{I.F.}) = \int Q(\text{I.F.})dy + c$.

Chap-18_Vector Algebra

Formulae Chart

1. **Vectors when co-ordinates of end points are given :** If $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ be the end-points of line segment AB, then $\overrightarrow{AB} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$.
   $(x_2 - x_1)$, $(y_2 - y_1)$ and $(z_2 - z_1)$ are components of $\overrightarrow{AB}$ along x, y and z-axis respectively. These are also direction ratios along x, y and z-axis respectively.
2. **Section formula :** Position vector $\vec{r}$ of a point which divides the join of two given points with position vector $\vec{a}$ and $\vec{b}$ in the ratios $l : m$ internally is $\vec{r} = \frac{l\vec{b} + m\vec{a}}{l + m}$
   Externally, $\vec{r} = \frac{l\vec{b} - m\vec{a}}{l - m}$
3. **Scalar product of two vectors :**
   (i) The scalar product of two vector $\vec{a}$ and $\vec{b}$ defined as $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$, where θ is the angle between $\vec{a}$ and $\vec{b}$.
   (ii) $\vec{a} \cdot \vec{b}$ is a scalar quantity.
   (iii) $\vec{a} \cdot \vec{b} = 0 \Leftrightarrow \vec{a} \perp \vec{b}$ (for non-zero vectors)
   (iv) For two vectors $\vec{a}$ and $\vec{b}$, $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$
   (v) For vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$, $\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$
   (vi) Projection of $\vec{a}$ along $\vec{b}$ is $\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$
   (vii) Angle between $\vec{a}$ and $\vec{b}$ is given by $\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$, θ is the angle between $\vec{a}$ and $\vec{b}$.
   (viii) If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, then $\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + a_3b_3$.
   If $\vec{a} \perp \vec{b} \Rightarrow a_1b_1 + a_2b_2 + a_3b_3 = 0$
4. **Vector or cross product :**
   (i) $\vec{a} \times \vec{b} = |\vec{a}||\vec{b}|\sin\theta\,\hat{n}$, θ is the angle between $\vec{a}$ and $\vec{b}$ and $\hat{n}$ is a unit vector perpendicular to $\vec{a}$ and $\vec{b}$ and direction is such that $\vec{a}$, $\vec{b}$ and $\hat{n}$ form a right hand system.

   (ii) $\vec{a} \times \vec{b}$ is a vector quantity whose magnitude is $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|\sin\theta$.
   (iii) If θ is angle between $\vec{a}$ and $\vec{b}$, then $\sin\theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|}$
   (iv) If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$
   Then $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$
   (v) Unit vector $\hat{n}$ perpendicular to $\vec{a}$ and $\vec{b}$ is $\hat{n} = \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$
5. **Scalar Triple product :**
   $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$
   $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$
   $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$
   $\vec{a} \cdot (\vec{b} \times \vec{c}) = [\vec{a}\,\vec{b}\,\vec{c}] = [\vec{b}\,\vec{c}\,\vec{a}] = [\vec{c}\,\vec{a}\,\vec{b}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$
   (i) If $\vec{a}$, $\vec{b}$, $\vec{c}$ are coplanar then $[\vec{a}\,\vec{b}\,\vec{c}] = 0$
   (ii) If $\vec{a}$, $\vec{b}$, $\vec{c}$ are coterminous edges of parallelepiped then volume of parallelepiped $= [\vec{a}\,\vec{b}\,\vec{c}]$.
   (iii) $\vec{a}$, $\vec{b}$, $\vec{c}$ are coterminous edges of tetrahedrons than volume of tetrahedrons $= \frac{1}{6}[\vec{a}\,\vec{b}\,\vec{c}]$.
6. **Vector Triple product :**
   $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$
   It gives a vector orthogonal to $\vec{a}$ and coplanar to $\vec{b}$ and $\vec{c}$.

Chap-19 3-D Geometry

Formulae Chart

1. **Shortest distance between for parallel lines**
   $\vec{r} = \vec{a} + \lambda\vec{b}$
   $\vec{r} = \vec{c} + \mu\vec{b}$
   is $\left|\frac{(\vec{a} - \vec{c}) \times \vec{b}}{|\vec{b}|}\right|$
2. **Shortest distance between two skew lines**
   $\vec{r} = \vec{a} + \lambda\vec{b}$
   $\vec{r} = \vec{c} + \mu\vec{d}$
   is $\left|\frac{(\vec{c} - \vec{a}) \cdot (\vec{b} \times \vec{d})}{|\vec{b} \times \vec{d}|}\right|$
3. **Condition for two lines to be coplanar or intersecting**
   $(\vec{c} - \vec{a}) \cdot (\vec{b} \times \vec{d}) = 0 \Rightarrow [\vec{c} - \vec{a}\;\vec{b}\;\vec{d}] = 0$
4. **Angle between line and plane**
   $L : \frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$
   $P : a_2x + b_2y + c_2z + d_2 = 0$
   $\sin\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$
   Line and plane are parallel if $a_1a_2 + b_1b_2 + c_1c_2 = 0$
   Line and planes are parallel if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
5. **Foot of perpendicular of point (α, β, γ) with respect to $ax + by + cz + d = 0$**
   $\frac{x_1 - \alpha}{a} = \frac{y_1 - \beta}{b} = \frac{z_1 - \gamma}{c} = -\frac{(a\alpha + b\beta + c\gamma + d)}{a^2 + b^2 + c^2}$, (x_1, y_1, z_1) is foot of perpendicular.
   Image of point (α, β, γ) with respect to $ax + by + cz + d = 0$ is
   $\frac{x_2 - \alpha}{a} = \frac{y_2 - \beta}{b} = \frac{z_2 - \gamma}{c} = \frac{-2(a\alpha + b\beta + c\gamma + d)}{a^2 + b^2 + c^2}$, (x_2, y_2, z_2) is image.

Chap-20 Probability

Formulae Chart

(i) Let S be a sample space and E be an event, then the probability of E, denoted by $P(E)$ is defined as $P(E) = \frac{n(E)}{n(S)}$
(ii) In case of some finite geometrical measurement such as length, area or volume, the probability of an event is defined as $P(E) = \frac{\text{Length of } E}{\text{Length of } S}$ or $\frac{\text{Area of } E}{\text{Area of } S}$ or $\frac{\text{Volume of } E}{\text{Volume of } S}$
(iii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
   $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$
   If A, B and C are mutually exclusive events, then
   $P(A \cup B \cup C) = P(A) + P(B) + P(C)$
   $P(A \cup B) = P(A) + P(B)$ and $P(A \cap B) = 0$
(iv) **Conditional Probability**
   If A and B are dependent events, then the probability of the event B when the event A has already occurred is called conditional probability denoted by $P\left(\frac{B}{A}\right)$ is defined as
   $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$, $P(A) \neq 0$
   Similarly $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$, $P(B) \neq 0$.
(v) **Total Probability**
   Let $E_1, E_2, E_3, \ldots\ldots\ldots E_n$ be n mutually exclusive and exhaustive events associated with a random experiment, then the probability of the occurrence of any event E, which occurs with E_1 or E_2 or E_3 .... or E_n is given by $P(E) = P(E_1)P\left(\frac{E}{E_1}\right) + P(E_2)P\left(\frac{E}{E_2}\right) + \ldots\ldots\ldots + P(E_n)P\left(\frac{E}{E_n}\right)$
(vi) **Baye's Theorem**
   Let $E_1, E_2, E_3, \ldots\ldots E_n$ be n mutually exclusive and exhaustive events associated with a random experiment and E is any other event which occurs with E_1 or E_2 or E_3 .... or E_n, then probability of the occurrence of the event E_1 or E_2 or E_3 .... or E_n when E has already occurred is given by

$$P\left(\frac{E_k}{E}\right) = \frac{P(E_k)P\left(\frac{E}{E_k}\right)}{\sum_{i=1}^{n} P(E_i)P\left(\frac{E}{E_i}\right)},\quad k = 1, 2, 3, \ldots\ldots, n$$

(vii) **Probabilities using Binomial Theorem**
   (a) Probability of k success in n trials
   $P(k) = {}^nC_k\,p^k q^{n-k}$, $k \leq n$
   (b) Probability of at least k success in n trials
   $P(\geq k) = \sum_{r=k}^{n} {}^nC_r\,p^r q^{n-r}$
   (c) Probability of atmost k success in n trials
   $P(\leq k) = \sum_{r=0}^{k} {}^nC_r\,p^r q^{n-r} = {}^nC_0\,p^0 q^n + {}^nC_1\,p^1 q^{n-1} + \ldots\ldots + {}^nC_k\,p^k q^{n-k}$
   (d) Probability of atleast one success
   $= 1 -$ Probability of number success $= 1 - {}^nC_0\,q^n$.
(viii) The mean, variance and standard deviation of binomial distribution are np, npq and $\sqrt{npq}$ respectively where
   n = Number of trials
   p = Probability of success (Occurence)
   q = Probability of failure (non-occurence)
(ix) The probability of selecting exactly one pair of shoes when $2r$ shoes are to be selected from n pairs of shoes is $\frac{n \cdot {}^{n-1}C_{2r-2}\,2^{2r-2}}{{}^{2n}C_{2r}}$

Content Courtesy:
Aakash Educational Services Ltd. New Delhi