2014

BOOKLET NO. TEST CODE : **MMA**

Forenoon

Questions : 30	**Time : 2 hours**

Write your Name, Registration Number, Test Centre, Test Code and the Number of this booklet in the appropriate places on the answersheet.

For each question, there are four suggested answers of which only one is correct. For each question indicate your choice of the correct answer by darkening the appropriate oval (⬬) completely on the answer sheet.

4 marks are allotted for each correct answer,
0 marks for each incorrect answer and
1 mark for each unattempted question.

ALL ROUGH WORK MUST BE DONE ON THIS BOOKLET ONLY.
YOU ARE NOT ALLOWED TO USE CALCULATORS IN ANY FORM.

STOP! WAIT FOR THE SIGNAL TO START.

1. The four distinct points $(-a, -b)$, $(0, 0)$, (a, b) and (a^2, ab) are

   (A) vertices of parallelogram (B) vertices of rectangle
   (C) collinear (D) lying on a circle.

2. Two integers m and n are chosen at random with replacement from the natural numbers $1, 2, \dots, 9$. The probability that $m^2 - n^2$ is divisible by 4 is

   (A) $\frac{41}{81}$ (B) $\frac{37}{81}$ (C) $\frac{2}{3}$ (D) $\frac{4}{9}$.

3. If $|2^z| = 1$ for a non-zero complex number z then which one of the following is necessarily true.

   (A) $Re(z) = 0$. (B) $|z| = 1$. (C) $Re(z) = 1$. (D) No such z exists.

4. How many pair of positive integers of (m, n) are there satisfying

$$\sum_{i=1}^{n} i! = m!$$

   (A) 0 (B) 1 (C) 2 (D) 3.

5. The value of $\lim_{x \to 0} \sin x \sin\left(\frac{1}{x}\right)$

   (A) is 0 (B) is 1 (C) is 2 (D) does not exist.

6. The sum of an infinite geometric series of real numbers is 14, and the sum of the cubes of the terms of this series is 392. Then the first term of the series is

   (A) -14 (B) 10 (C) 7 (D) -5.

7. The value of the integral $\int_0^\pi \frac{x}{1+\sin^2 x} dx$ is

   (A) $2\sqrt{2}\pi^2$ (B) $\frac{\pi^2}{2\sqrt{2}}$ (C) $\frac{\pi^2}{\sqrt{2}}$ (D) $\sqrt{2}\pi^2$.

8. Number of integers x between 1 and 95 such that 96 divides $60x$ is

(A) 0 (B) 7 (C) 8 (D) 11.

9. Consider the function for all $x \in (0, 1)$,

$$f(x) = \begin{cases} x & \text{if } x \text{ is rational,} \\ 1 - x & \text{otherwise.} \end{cases}$$

Then, the function f is

(A) everywhere continuous on $(0, 1)$
(B) continuous only on rational points in $(0, 1)$
(C) nowhere continuous on $(0, 1)$
(D) continuous only at a single point in $(0, 1)$.

10. E_1 and E_2 are two events such that $P(E_1) = 0.2$ and $P(E_2) = 0.5$. What are the minimum and maximum possible values of $P(E_1^c \mid E_2^c)$?

(A) 0 and 0.6 (B) 0.4 and 0.6 (C) 0.4 and 1 (D) 0.6 and 1.

11. Let X_1, X_2, X_3, X_4 be i.i.d. random variables each assuming the values 1 and -1 with probability $1/2$ each. Then, the probability that the matrix

$$\begin{pmatrix} X_1 & X_2 \\ X_3 & X_4 \end{pmatrix}$$

is nonsingular equals

(A) $1/2$ (B) $3/8$ (C) $5/8$ (D) $1/4$.

12. For any matrix A, let A^t denote its transpose matrix. What is the minimum value of $\mathrm{trace}(AA^t)$ for an $n \times n$ nonsingular matrix A with integer entries?

(A) 0 (B) 1 (C) n (D) n^2.

13. The area bounded by the curves $arg(z) = \pi/3, arg(z) = 2\pi/3$ and $arg(z - 2 - 2\sqrt{3}i) = \pi$ on the complex plane is given by

(A) $2\sqrt{3}$ (B) $4\sqrt{3}$ (C) $\sqrt{3}$ (D) $3\sqrt{3}$.

14. The locus of the center of a circle that passes through origin and cuts off a length $2a$ from the line $y = c$ is

(A) $x^2 + 2cx = a^2 + c^2$ (B) $x^2 + 2cy = a^2 + c^2$

(C) $y^2 + cx = a^2 + c^2$ (D) $y^2 + 2cy = a^2 + c^2$.

15. Let $f : \mathbb{R} \to \mathbb{R}$ be such that $f(x) = x^3 - 3x^2 + 5x - 10$. Then, f is

(A) neither one to one nor onto

(B) one to one but not onto

(C) not one to one but onto

(D) both one to one and onto.

16. Let X, Y be random variables such that

$$P(X = 1 \mid Y = 1) = P(X = 1 \mid Y = 2) = P(X = 1 \mid Y = 3) = \frac{1}{2}.$$

Then, which one of the following statements is true?

(A) No such X and Y can exist satisfying the above condition.

(B) $P(X = 1 \mid Y \in \{1, 2, 3\}) = 1/2$.

(C) $P(X = 1 \mid Y \in \{1, 2, 3\}) < 1/2$.

(D) $P(X = 1 \mid Y \in \{1, 2, 3\}) > 1/2$.

17. What is the minimum value of $|z + w|$ for complex numbers z and w with $zw = 1$?

(A) 0 (B) 1 (C) 2 (D) 3.

18. Let

$$D_1 = \mathbf{det}\begin{pmatrix} a & b & c \\ x & y & z \\ p & q & r \end{pmatrix} \text{ and } D_2 = \mathbf{det}\begin{pmatrix} -x & a & -p \\ y & -b & q \\ z & -c & r \end{pmatrix}.$$

Then

(A) $D_1 = D_2$ (B) $D_1 = 2D_2$ (C) $D_1 = -D_2$ (D) $D_2 = 2D_1$.

19. Let a and b be real numbers such that

$$\lim_{x \to 0} \left(\frac{\sin 3x}{x^3} + \frac{a}{x^2} + b \right) = 0.$$

Then

(A) $a = -3, \ b = 9/2$ (B) $a = 3, \ b = 9/2$

(C) $a = -3, \ b = -9/2$ (D) $a = -3, \ b = -9/2$.

20. Let α, β, γ be the roots of $x^3 - px + q = 0$. Then

$$\mathbf{det} \begin{pmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{pmatrix} \text{ equals to}$$

(A) p (B) q (C) pq (D) 0.

21. Let x, y, z be consecutive positive integers such that $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} > \frac{1}{10}$. Then the maximum value of $x + y + z$ is

(A) 99 (B) 96 (C) 90 (D) 87.

22. Let G denote the group $\{1, \omega, \omega^2\}$ under multiplication where ω is a complex cube root of unity. How many group homomorphism are there from G to S_3, the permutation group over 3 elements?

(A) 1 (B) 2 (C) 3 (D) 6.

23. Let $1, w_1, w_2, \ldots, w_9$ be the distinct complex 10^{th} roots of unity. Then

$$(1 - w_1) \cdots (1 - w_9) \times (\sum_{j=1}^{9} \frac{1}{1 - w_j}) \quad \text{equals to}$$

(A) 90 (B) 45 (C) 10 (D) 9.

24. The number of five digit integers of the form $x678y$ which is divisible by 55 is

(A) 0 (B) 1 (C) 2 (D) 4.

25. Let $[x]$ denote the greatest integer less than or equal to x. Then

$$\lim_{n\to\infty} \sqrt{n^2+2n} - [\sqrt{n^2+2n}]$$

(A) is 0 (B) is 1/2 (C) is 1 (D) does not exist.

26. Let $f : (0,\infty) \to \mathbb{R}$ be a function defined as $f(x) = x^{1/x}$ then the range of f is

(A) $(0,\infty)$ (B) $(0,1]$ (C) $(0,e^{1/e}]$ (D) $[e^{1/e},\infty)$.

27. A sequence of real numbers x_n are defined as follows:

$$x_{n+2} = \frac{1+x_{n+1}}{x_n},\ n = 0,1,2,\cdots \text{ and } x_0 = 1,\ x_1 = 2.$$

Then x_{2014} equals to

(A) 1 (B) 2 (C) 3 (D) none of the above.

28. How many triangles can be formed with vertices of a 10-sided polygon so that no side of the triangle is a side of the polygon?

(A) $\binom{10}{3}$ (B) $\binom{8}{3}$ (C) $\binom{10}{3} - 80$ (D) $\binom{10}{3} - 70$.

29. Let $S = \{1,2,\ldots,10\}$. Then the number of pairs (A,B), where A and B are non-empty disjoint subsets of S is

(A) $3^{10}-1$ (B) $3^{10}-2^{10}$ (C) $3^{10}-2^{10}+1$ (D) $3^{10}-2^{11}+1$.

30. Let $\mathbf{v}^t$ denote the transpose of the column vector $\mathbf{v}$. Consider two non-zero p-dimensional column vectors $\mathbf{a}$ and $\mathbf{b}$, $p \geq 2$. How many non-zero distinct eigenvalues does the $p \times p$ matrix $\mathbf{a}\mathbf{b}^t + \mathbf{b}\mathbf{a}^t$ have?

(A) 0 or 1 (B) 1 or 2 (C) exactly 2 (D) 0, 1, or 2.