READ THE INSTRUCTIONS CAREFULLY

GENERAL

- 1. This sealed booklet is your Question Paper. Do not break the seal till you are told to do so.
- The paper CODE is printed on the right hand top corner of this sheet and the right hand top corner of the back cover of this booklet.
- 3. Use the Optical Response Sheet (ORS) provided separately for answering the questions.
- 4. The paper CODE is printed on the left part as well as the right part of the ORS. Ensure that both these codes are identical and same as that on the question paper booklet. If not, contact the invigilator for change of ORS.
- 5. Blank spaces are provided within this booklet for rough work.
- 6. Write your name, roll number and sign in the space provided on the back cover of this booklet.
- After breaking the seal of the booklet at 2:00 pm, verify that the booklet contains 36 pages and that all the 54 questions along with the options are legible. If not, contact the invigilator for replacement of the booklet.
- You are allowed to take away the Question Paper at the end of the examination.

OPTICAL RESPONSE SHEET

- The ORS (top sheet) will be provided with an attached Candidate's Sheet (bottom sheet).The Candidate's Sheet is a carbon-less copy of the ORS.
- Darken the appropriate bubbles on the ORS by applying sufficient pressure. This will leave an impression at the corresponding place on the Candidate's Sheet.
- 11. The ORS will be collected by the invigilator at the end of the examination...
- 12. You will be allowed to take away the Candidate's Sheet at the end of the examination.
- 13. Do not tamper with or mutilate the ORS. Do not use the ORS for rough work.
- 14. Write your name, roll number and code of the examination center, and sign with pen in the space provided for this purpose on the ORS. Do not write any of these details anywhere else on the ORS. Darken the appropriate bubble under each digit of your roll number.

DARKENING THE BUBBLES ON THE ORS

- 15. Use a BLACK BALL POINT PEN to darken the bubbles on the ORS.
- 16. Darken the bubble COMPLETELY.
- 17. The correct way of darkening a bubble is as:
- 18. The ORS is machine-gradable. Ensure that the bubbles are darkened in the correct way.
- 19. Darken the bubbles ONLY IF you are sure of the answer. There is NO WAY to erase or "un-darken" a darkened bubble.

Please see the last page of this booklet for rest of the instructions.



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PART I: PHYSICS

SECTION 1 (Maximum Marks: 21)

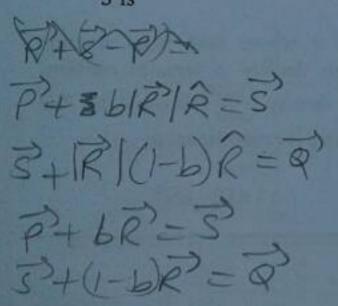
- · This section contains SEVEN questions
- Each question has FOUR options [A], [B], [C] and [D]. ONLY ONE of these four options is
- For each question, darken the bubble corresponding to the correct option in the ORS
- · For each question, marks will be awarded in one of the following categories:

Full Marks : +3 If only the bubble corresponding to the correct option is darkened

Zero Marks : 0 If none of the bubbles is darkened

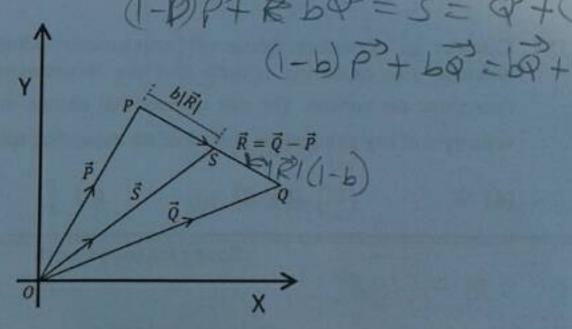
Negative Marks: -1 In all other cases

Q.1 Three vectors \vec{P} , \vec{Q} and \vec{R} are shown in the figure. Let S be any point on the vector \vec{R} . The distance between the points P and S is $b|\vec{R}|$. The general relation among vectors \vec{P} , \vec{Q} and \vec{S} is



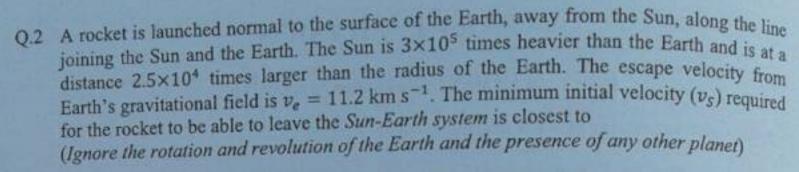
[A] $\vec{S} = (1-b)\vec{P} + b^2\vec{Q}$

[C]
$$\vec{S} = (1 - b^2)\vec{P} + b\vec{Q}$$



 $[B] \vec{S} = (1-b)\vec{P} + b\vec{Q}$

[D] $\vec{S} = (b-1)\vec{P} + b\vec{Q}$



[A]
$$v_S = 72 \text{ km s}^{-1}$$
 [B] $v_S = 22 \text{ km s}^{-1}$ [C] $v_S = 62 \text{ km s}^{-1}$

A person measures the depth of a well by measuring the time interval between dropping a stone and receiving the sound of impact with the bottom of the well. The error in his measurement of time is $\delta T = 0.01$ seconds and he measures the depth of the well to be L = 20 meters. Take the acceleration due to gravity g = 10 ms⁻² and the velocity of sound is 300 ms⁻¹. Then the fractional error in the measurement, $\delta L/L$, is closest to

Q.4 Consider an expanding sphere of instantaneous radius R whose total mass remains constant. The expansion is such that the *instantaneous* density ρ remains uniform throughout the volume. The rate of fractional change in density $\left(\frac{1}{\rho}\frac{d\rho}{dt}\right)$ is constant. The velocity ν of any point on the surface of the expanding sphere is proportional to

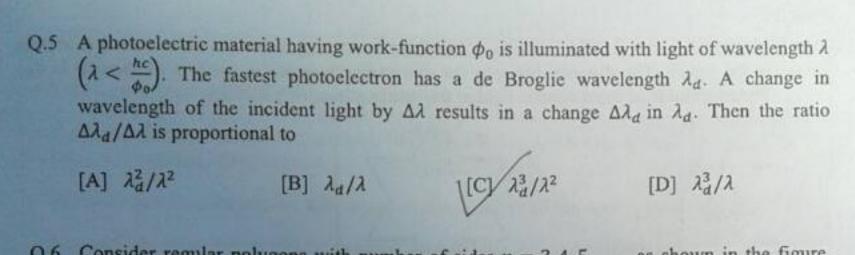
$$V_{C} = \sqrt{2gR} \qquad Space for rough work$$

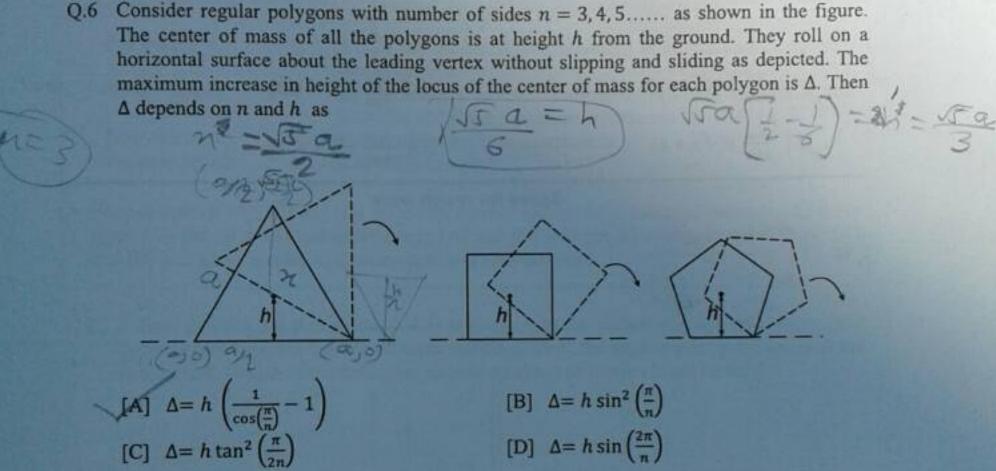
$$V_{C} = \sqrt{2gR} \qquad Space for rough work$$

$$-\frac{GM_{M}}{R} + -\frac{G(3\times10^{8})M_{M}}{3^{5}} + \frac{1}{2} \frac{11.2}{M} = 0$$

$$V^{2} = 266 \qquad 26M \qquad 1+12 \qquad 11.2 \qquad \times 3.7$$

$$V^{3} = \sqrt{3} \times V_{C} \qquad 1+12 \qquad M = \frac{4}{3} + \sqrt{3} + \sqrt$$

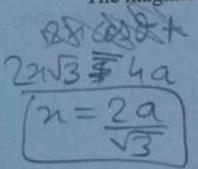


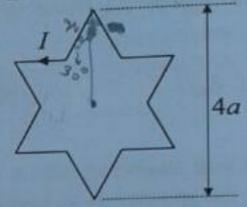


Space for rough work

$$\frac{h^2}{2m\Lambda^2} = \frac{h^2}{2m\Lambda^2} = \frac{h^2$$

Q.7 A symmetric star shaped conducting wire loop is carrying a steady state current I as shown in the figure. The distance between the diametrically opposite vertices of the star is 4a. The magnitude of the magnetic field at the center of the loop is





[A]
$$\frac{\mu_0 I}{4\pi a} 3[\sqrt{3} - 1]$$

$$[C] \frac{\mu_0 I}{4\pi a} 6[\sqrt{3} + 1]$$

[B]
$$\frac{\mu_0 I}{4\pi a} 3[2-\sqrt{3}]$$

[D]
$$\frac{\mu_0 l}{4\pi a} 6[\sqrt{3} - 1]$$

SECTION 2 (Maximum Marks: 28)

- This section contains SEVEN questions
- Each question has FOUR options [A], [B], [C] and [D]. ONE OR MORE THAN ONE of these four options is(are) correct
- · For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS
- · For each question, marks will be awarded in one of the following categories:

Full Marks : +4 If only the bubble(s) corresponding to all the correct option(s)

is(are) darkened

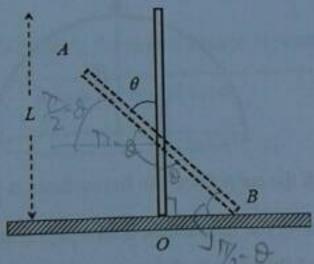
Partial Marks : +1 For darkening a bubble corresponding to each correct option,

provided NO incorrect option is darkened

Zero Marks : 0 If none of the bubbles is darkened

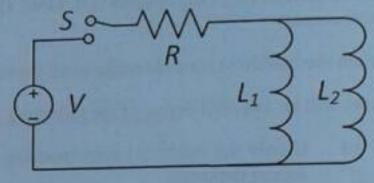
Negative Marks : -2 In all other cases

- For example, if [A], [C] and [D] are all the correct options for a question, darkening all these
 three will get +4 marks; darkening only [A] and [D] will get +2 marks; and darkening [A]
 and [B] will get -2 marks, as a wrong option is also darkened
- Q.8 A rigid uniform bar AB of length L is slipping from its vertical position on a frictionless floor (as shown in the figure). At some instant of time, the angle made by the bar with the vertical is θ . Which of the following statements about its motion is/are correct?

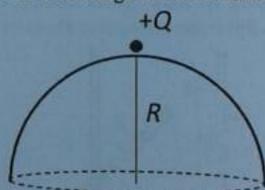


- When the bar makes an angle θ with the vertical, the displacement of its midpoint from the initial position is proportional to $(1 \cos \theta)$
- [B] The midpoint of the bar will fall vertically downward
 - [C] The trajectory of the point A is a parabola
 - [D] Instantaneous torque about the point in contact with the floor is proportional to $\sin \theta$

Q.9 A source of constant voltage V is connected to a resistance R and two ideal inductors L_1 and L_2 through a switch S as shown. There is no mutual inductance between the two inductors. The switch S is initially open. At t=0, the switch is closed and current begins to flow. Which of the following options is/are correct?

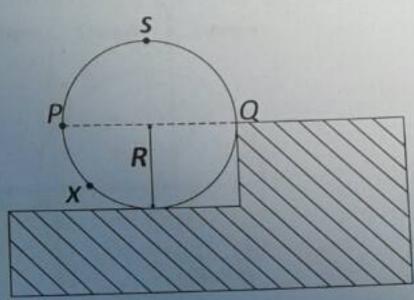


- After a long time, the current through L_2 will be $\frac{v}{R} \frac{L_1}{L_1 + L_2}$
- [B] The ratio of the currents through L_1 and L_2 is fixed at all times (t > 0)
- [C] After a long time, the current through L_1 will be $\frac{V}{R} \frac{L_2}{L_1 + L_2}$
 - [D] At t = 0, the current through the resistance R is $\frac{V}{R}$
- Q.10 A point charge +Q is placed just outside an imaginary hemispherical surface of radius R as shown in the figure. Which of the following statements is/are correct?

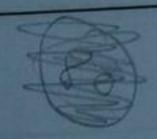


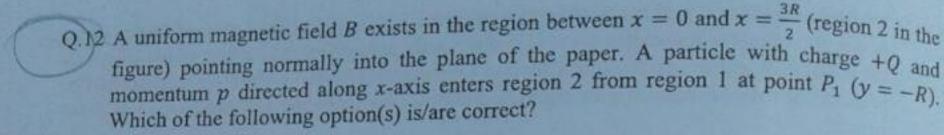
- [A] Total flux through the curved and the flat surfaces is $\frac{Q}{\epsilon_0}$
- [B] The circumference of the flat surface is an equipotential
 - [C] The component of the electric field normal to the flat surface is constant over the surface
- The electric flux passing through the curved surface of the hemisphere is $-\frac{Q}{2\varepsilon_0}\left(1-\frac{1}{\sqrt{2}}\right)$

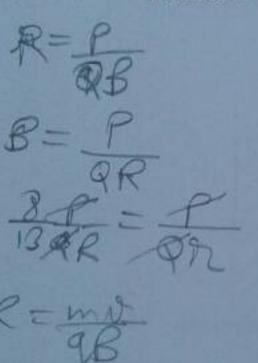
Q.11 A wheel of radius R and mass M is placed at the bottom of a fixed step of height R as shown in the figure. A constant force is continuously applied on the surface of the wheel so that it just climbs the step without slipping. Consider the torque τ about an axis normal to the plane of the paper passing through the point Q. Which of the following options is/are correct?

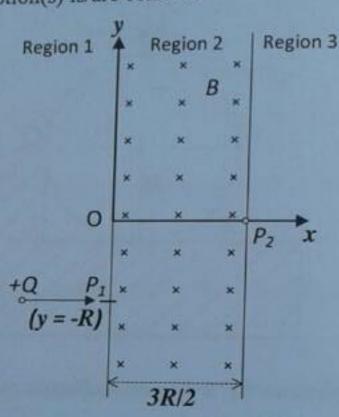


- [A] If the force is applied normal to the circumference at point X then τ is constant
- [B] If the force is applied at point P tangentially then τ decreases continuously as the wheel climbs
 - If the force is applied tangentially at point S then $\tau \neq 0$ but the wheel never climbs the step
- [D] If the force is applied normal to the circumference at point P then τ is zero





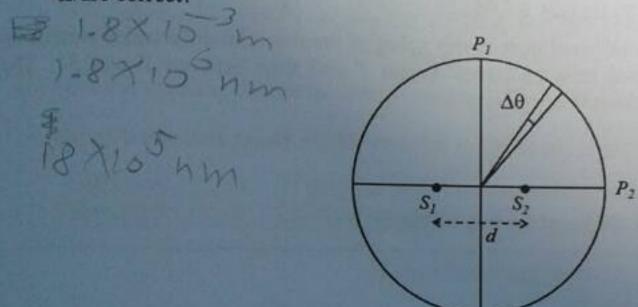




- [A] For $B = \frac{8}{13} \frac{p}{QR}$, the particle will enter region 3 through the point P_2 on x-axis
- When the particle re-enters region 1 through the longest possible path in region 2, the magnitude of the change in its linear momentum between point P_1 and the farthest point from y-axis is $p/\sqrt{2}$
 - [C] For a fixed B, particles of same charge Q and same velocity v, the distance between the point P_1 and the point of re-entry into region 1 is inversely proportional to the mass of the particle

[D] For $B > \frac{2}{3} \frac{p}{QR}$, the particle will re-enter region 1

Q.13 Two coherent monochromatic point sources S_1 and S_2 of wavelength $\lambda = 600$ nm are placed symmetrically on either side of the center of the circle as shown. The sources are separated by a distance d = 1.8 mm. This arrangement produces interference fringes visible as alternate bright and dark spots on the circumference of the circle. The angular separation between two consecutive bright spots is $\Delta\theta$. Which of the following options is/are correct?



- [A] A dark spot will be formed at the point P_2
- [B] At P_2 the order of the fringe will be maximum
- [C] The angular separation between two consecutive bright spots decreases as we move from P_1 to P_2 along the first quadrant
- [D] The total number of fringes produced between P_1 and P_2 in the first quadrant is close to 3000

Q.14 The instantaneous voltages at three terminals marked X, Y and Z are given by

$$V_X = V_0 \sin \omega t$$
,
 $V_Y = V_0 \sin \left(\omega t + \frac{2\pi}{3}\right)$ and
 $V_Z = V_0 \sin \left(\omega t + \frac{4\pi}{3}\right)$.

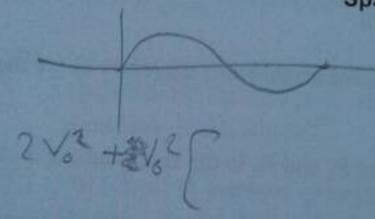
An ideal voltmeter is configured to read rms value of the potential difference between its terminals. It is connected between points X and Y and then between Y and Z. The reading(s) of the voltmeter will be

$$[A] V_{XY}^{rms} = V_0 \sqrt{\frac{3}{2}}$$

independent of the choice of the two terminals

[C]
$$V_{YZ}^{rms} = V_0 \sqrt{\frac{1}{2}}$$

[D]
$$V_{XY}^{rms} = V_0$$



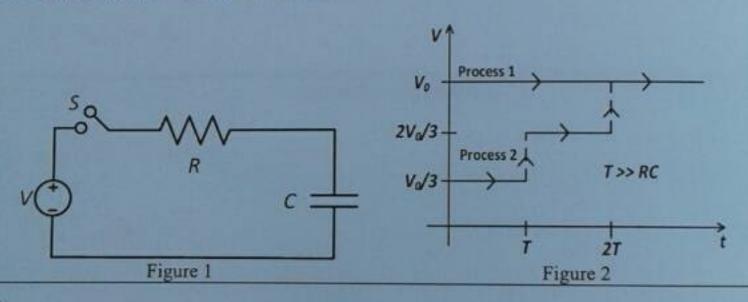
PARAGRAPH 1

Consider a simple RC circuit as shown in Figure 1.

Process 1: In the circuit the switch S is closed at t = 0 and the capacitor is fully charged to voltage V_0 (i.e., charging continues for time T >> RC). In the process some dissipation (E_D) occurs across the resistance R. The amount of energy finally stored in the fully charged capacitor is E_C .

Process 2: In a different process the voltage is first set to $\frac{v_0}{3}$ and maintained for a charging time T >> RC. Then the voltage is raised to $\frac{2V_0}{3}$ without discharging the capacitor and again maintained for a time T >> RC. The process is repeated one more time by raising the voltage to V_0 and the capacitor is charged to the same final voltage V_0 as in Process 1.

These two processes are depicted in Figure 2.



Q.15 In Process 1, the energy stored in the capacitor E_C and heat dissipated across resistance E_D are related by:

[A]
$$E_C = E_D \ln 2$$

[B]
$$E_C = 2E_D$$

$$[C]$$
 $E_C = E_D$

[B]
$$E_C = 2E_D$$
 [C]
[D] $E_C = \frac{1}{2}E_D$

Q.16 In Process 2, total energy dissipated across the resistance E_D is:

[A]
$$E_D = \frac{1}{2}CV_0^2$$

[B]
$$E_D = 3 CV_0^2$$

[C]
$$E_D = \frac{1}{3} \left(\frac{1}{2} C V_0^2 \right)$$

[D]
$$E_D = 3\left(\frac{1}{2}CV_0^2\right)$$

PARAGRAPH 2

One twirls a circular ring (of mass M and radius R) near the tip of one's finger as shown in Figure 1. In the process the finger never loses contact with the inner rim of the ring. The finger traces out the surface of a cone, shown by the dotted line. The radius of the path traced out by the point where the ring and the finger is in contact is r. The finger rotates with an angular velocity ω_0 . The rotating ring rolls without slipping on the outside of a smaller circle described by the point where the ring and the finger is in contact (Figure 2). The coefficient of friction between the ring and the finger is μ and the acceleration due to gravity is g.

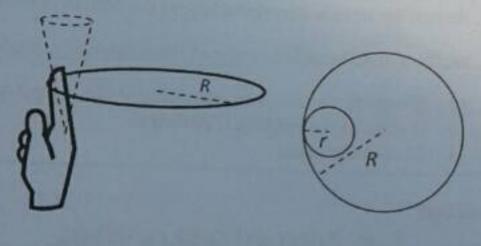


Figure 1

Figure 2

Q.17 The total kinetic energy of the ring is

[A]
$$\frac{1}{2}M\omega_0^2(R-r)^2$$

[B]
$$\frac{3}{2}M\omega_0^2(R-r)^2$$

[C]
$$M\omega_0^2(R-r)^2$$

[D]
$$M\omega_0^2 R^2$$

Q.18 The minimum value of ω_0 below which the ring will drop down is

[A]
$$\sqrt{\frac{2g}{\mu(R-r)}}$$

[A]
$$\sqrt{\frac{2g}{\mu(R-r)}}$$
 [B] $\sqrt{\frac{3g}{2\mu(R-r)}}$ [C] $\sqrt{\frac{g}{\mu(R-r)}}$ [D] $\sqrt{\frac{g}{2\mu(R-r)}}$

[C]
$$\sqrt{\frac{g}{\mu(R-r)}}$$

[D]
$$\sqrt{\frac{g}{2\mu(R-r)}}$$

END OF PART I: PHYSICS

Space for rough work

Sprage (R-2) g

PART II: CHEMISTRY

SECTION 1 (Maximum Marks: 21)

- This section contains SEVEN questions
- Each question has FOUR options [A], [B], [C] and [D]. ONLY ONE of these four options is
 correct
- · For each question, darken the bubble corresponding to the correct option in the ORS
- · For each question, marks will be awarded in one of the following categories:

Full Marks : +3 If only the bubble corresponding to the correct option is darkened

Zero Marks : 0 If none of the bubbles is darkened

Negative Marks: -1 In all other cases

Q.19 For the following cell,

$$Zn(s) \mid ZnSO_4(aq) \mid CuSO_4(aq) \mid Cu(s)$$

when the concentration of Zn^{2+} is 10 times the concentration of Cu^{2+} , the expression for ΔG (in J mol⁻¹) is

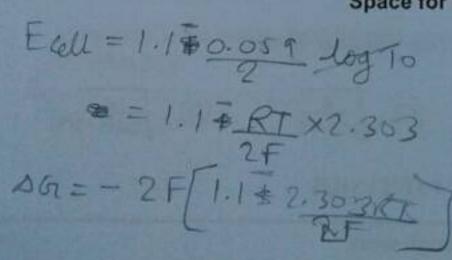
[F is Faraday constant; R is gas constant; T is temperature; $E^{\circ}(cell) = 1.1 \text{ V}$]

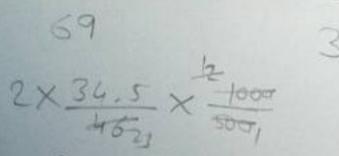
[A] 2.303RT - 2.2F

[B] -2.2F

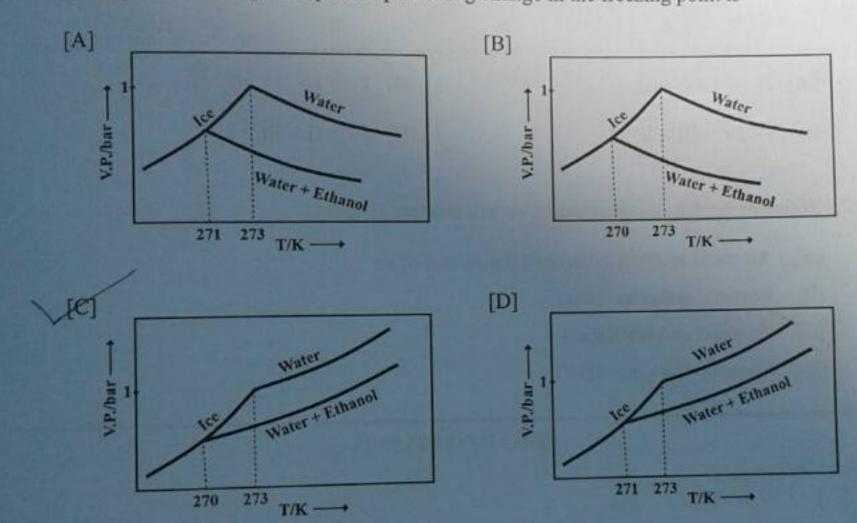
[C] 1.1F

[D] 2.303RT + 1.1F





Q.20 Pure water freezes at 273 K and 1 bar. The addition of 34.5 g of ethanol to 500 g of water changes the freezing point of the solution. Use the freezing point depression constant of water as 2 K kg mol-1. The figures shown below represent plots of vapour pressure (V.P.) versus temperature (T). [molecular weight of ethanol is 46 g mol-1] Among the following, the option representing change in the freezing point is



Q.21 The standard state Gibbs free energies of formation of C(graphite) and C(diamond) at DG1 = 0 T = 298 K are

 $\Delta_f G^a[C(graphite)] = 0 \text{ kJ mol}^{-1}$ $\Delta_f G^o[C(\text{diamond})] = 2.9 \text{ kJ mol}^{-1}$.

The standard state means that the pressure should be 1 bar, and substance should be pure at a given temperature. The conversion of graphite [C(graphite)] to diamond [C(diamond)] reduces its volume by 2×10⁻⁶ m³ mol⁻¹. If C(graphite) is converted to C(diamond) isothermally at T = 298 K, the pressure at which C(graphite) is in equilibrium with

[Useful information: $1 J = 1 \text{ kg m}^2 \text{ s}^{-2}$; $1 Pa = 1 \text{ kg m}^{-1} \text{ s}^{-2}$; $1 \text{ bar} = 10^5 \text{ Pa}$]

[A] 29001 bar

[B] 58001 bar [C] 14501 bar

[D] 1450 bar

Q.22 The order of basicity among the following compounds is

- Q.23 Which of the following combination will produce H2 gas?
 - [A] Au metal and NaCN(aq) in the presence of air
 - [B] Fe metal and conc. HNO₃
 - [E] Zn metal and NaOH(aq)
 - [D] Cu metal and conc. HNO₃

Space for rough work

Vaon-> Nazznoz+ Hz

Q.24 The order of the oxidation state of the phosphorus atom in H₃PO₂, H₃PO₄, H₃PO₃, and H₄P₂O₆ is

[A] H₃PO₄> H₃PO₂> H₃PO₃> H₄P₂O₆

[B] $H_3PO_3 > H_3PO_2 > H_3PO_4 > H_4P_2O_6$

[C] H₃PO₄ > H₄P₂O₆ > H₃PO₃ > H₃PO₂

[D] $H_3PO_2 > H_3PO_3 > H_4P_2O_6 > H_3PO_4$

Q.25 The major product of the following reaction is

SECTION 2 (Maximum Marks: 28)

- · This section contains SEVEN questions
- Each question has FOUR options [A], [B], [C] and [D]. ONE OR MORE THAN ONE of these four options is(are) correct
- · For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS
- · For each question, marks will be awarded in one of the following categories:

Full Marks : +4 If only the bubble(s) corresponding to all the correct option(s)

is(are) darkened

Partial Marks : +1 For darkening a bubble corresponding to each correct option,

provided NO incorrect option is darkened

Zero Marks : 0 If none of the bubbles is darkened

Negative Marks : -2 In all other cases

- For example, if [A], [C] and [D] are all the correct options for a question, darkening all these
 three will get +4 marks; darkening only [A] and [D] will get +2 marks; and darkening [A]
 and [B] will get -2 marks, as a wrong option is also darkened
- Q.26 Among the following, the correct statement(s) is(are)
 - [A] Al(CH₃)₃ has the three-centre two-electron bonds in its dimeric structure
 - [B] AlCl₃ has the three-centre two-electron bonds in its dimeric structure
 - [C] BH3 has the three-centre two-electron bonds in its dimeric structure
 - [D] The Lewis acidity of BCl3 is greater than that of AlCl3

RMK= AS-AT

- Q.27 For a reaction taking place in a container in equilibrium with its surroundings, the effect of temperature on its equilibrium constant K in terms of change in entropy is described by
 - [A] With increase in temperature, the value of K for exothermic reaction decreases because favourable change in entropy of the surroundings decreases
 - [B] With increase in temperature, the value of K for endothermic reaction increases because the entropy change of the system is negative
 - [C] With increase in temperature, the value of K for endothermic reaction increases because unfavourable change in entropy of the surroundings decreases
 - [D] With increase in temperature, the value of K for exothermic reaction decreases because the entropy change of the system is positive

Q.28 The option(s) with only amphoteric oxides is(are)

[A] Cr₂O₃, BeO, SnO, SnO₂

[B] ZnO, Al₂O₃, PbO, PbO₂

[C] NO, B₂O₃, PbO, SnO₂

[D] Cr₂O₃, CrO, SnO, PbO

- Q.29 In a bimolecular reaction, the steric factor P was experimentally determined to be 4.5. The correct option(s) among the following is(are)
 - [A] The activation energy of the reaction is unaffected by the value of the steric factor
 - [B] The value of frequency factor predicted by Arrhenius equation is higher than that determined experimentally
 - [C] Since P = 4.5, the reaction will not proceed unless an effective catalyst is used
 - [D] Experimentally determined value of frequency factor is higher than that predicted by Arrhenius equation

Q.30 For the following compounds, the correct statement(s) with respect to nucleophilic

- [A] Compound IV undergoes inversion of configuration
- [B] The order of reactivity for I, III and IV is: IV > I > III
- [e] I and III follow S_N1 mechanism
- [D] I and II follow S_N2 mechanism

Q.31 Compounds P and R upon ozonolysis produce Q and S, respectively. The molecular formula of Q and S is C₈H₈O. Q undergoes Cannizzaro reaction but not haloform reaction, whereas S undergoes haloform reaction but not Cannizzaro reaction.

(i) P
$$\frac{i) O_3/CH_2Cl_2}{ii) Zn/H_2O}$$
 Q (C_8H_8O)

(ii) R
$$\frac{i) O_3/CH_2Cl_2}{ii) Zn/H_2O}$$
 S (C_8H_8O)

The option(s) with suitable combination of P and R, respectively, is(are)

- Q.32 The correct statement(s) about surface properties is(are)
 - The critical temperatures of ethane and nitrogen are 563 K and 126 K, respectively. The adsorption of ethane will be more than that of nitrogen on same amount of activated charcoal at a given temperature
 - Adsorption is accompanied by decrease in enthalpy and decrease in entropy of the system
 - [C] Brownian motion of colloidal particles does not depend on the size of the particles but depends on viscosity of the solution
 - Cloud is an emulsion type of colloid in which liquid is dispersed phase and gas is dispersion medium

SECTION 3 (Maximum Marks: 12)

- · This section contains TWO paragraphs
- · Based on each paragraph, there are TWO questions
- Each question has FOUR options [A], [B], [C], and [D]. ONLY ONE of these four options
 is correct
- For each question, darken the bubble corresponding to the correct option in the ORS
- · For each question, marks will be awarded in one of the following categories:

Full Marks : +3 If only the bubble corresponding to the correct option is darkened

Zero Marks: 0 In all other cases

PARAGRAPH 1

Upon heating KClO₃ in the presence of catalytic amount of MnO₂, a gas W is formed. Excess amount of W reacts with white phosphorus to give X. The reaction of X with pure HNO₃ gives Y and Z.

Q.33 W and X are, respectively

[A] O2 and P4O6

[C] O₃ and P₄O₆

[B] O₃ and P₄O₁₀

[D] O₂ and P₄O₁₀

Q.34 Y and Z are, respectively

[A] N₂O₄ and HPO₃

[C] N₂O₅ and HPO₃

[B] N₂O₃ and H₃PO₄

[D] N₂O₄ and H₃PO₃

Space for rough work

Py 910 HATHNO3

PARAGRAPH 2

The reaction of compound P with CH₃MgBr (excess) in (C₂H₅)₂O followed by addition of H₂O CH₃COCl in the presence of anhydrous AlCl₃ in CH₂Cl₂ followed by treatment with H₂O produces compound S. [Et in compound P is ethyl group]

$$(H_3C)_3C$$
 CO_2Et
 $Q \longrightarrow R \longrightarrow S$

Q.35 The reactions, Q to R and R to S, are

- [A] Dehydration and Friedel-Crafts acylation
 - [B] Friedel-Crafts alkylation, dehydration and Friedel-Crafts acylation
 - [C] Aromatic sulfonation and Friedel-Crafts acylation
 - [D] Friedel-Crafts alkylation and Friedel-Crafts acylation

Q.36 The product S is

[C]
$$(H_3C)_3C$$
 $(H_3C)_3C$ $(H_3C)_3C$

END OF PART II : CHEMISTRY

PART III: MATHEMATICS

SECTION 1 (Maximum Marks: 21)

- · This section contains SEVEN questions
- Each question has FOUR options [A], [B], [C] and [D]. ONLY ONE of these four options is correct
- · For each question, darken the bubble corresponding to the correct option in the ORS
- · For each question, marks will be awarded in one of the following categories:

Full Marks : +3 If only the bubble corresponding to the correct option is darkened

Zero Marks : 0 If none of the bubbles is darkened

Negative Marks: -1 In all other cases

Q.37 The equation of the plane passing through the point (1, 1, 1) and perpendicular to the planes 2x + y - 2z = 5 and 3x - 6y - 2z = 7, is

[A]
$$-14x + 2y + 15z = 3$$

[B]
$$14x + 2y + 15z = 31$$

[C]
$$14x + 2y - 15z = 1$$

[D]
$$14x - 2y + 15z = 27$$

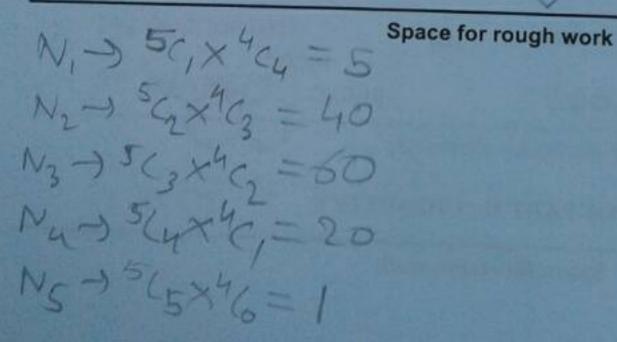
Q.38 Let $S = \{1, 2, 3, ..., 9\}$. For k = 1, 2, ..., 5, let N_k be the number of subsets of S, each containing five elements out of which exactly k are odd. Then $N_1 + N_2 + N_3 + N_4 + N_5 =$

[A] 252

[B] 125

[C] 126

[D] 210



Q.39 If $f: \mathbb{R} \to \mathbb{R}$ is a twice differentiable function such that f''(x) > 0 for all $x \in \mathbb{R}$, and $f\left(\frac{1}{2}\right) = \frac{1}{2}$, f(1) = 1, then

[B]
$$f'(1) \le 0$$

[C]
$$\frac{1}{2} < f'(1) \le 1$$

[D]
$$0 < f'(1) \le \frac{1}{2}$$

Q.40 Let O be the origin and let PQR be an arbitrary triangle. The point S is such that

$$\overrightarrow{OP} \cdot \overrightarrow{OQ} + \overrightarrow{OR} \cdot \overrightarrow{OS} = \overrightarrow{OR} \cdot \overrightarrow{OP} + \overrightarrow{OQ} \cdot \overrightarrow{OS} = \overrightarrow{OQ} \cdot \overrightarrow{OR} + \overrightarrow{OP} \cdot \overrightarrow{OS}$$

Then the triangle PQR has S as its

[A] circumcentre

[B] incentre

[C] centroid

[D] orthocenter

Space for rough work

P

SP = 0, R = ar, op = 2?

A

SP = anity;

O + ania
2 + 0 = otan;

ota

Q.41 How many diagonal ent	3×3 matrices M with entries tries of M^TM is 5?	s from {0, 1, 2} are th	ere, for which the sum	of the
[A] 198	[B] 135	[C] 126	[D] 162	
Q.42 If y = y(x)	satisfies the differential equ	ation		
	$8\sqrt{x}\left(\sqrt{9+\sqrt{x}}\right)dy = \left(\sqrt{\frac{9+\sqrt{x}}{x}}\right)dy = \left(\sqrt{\frac{9+\sqrt{x}}{x}}\right)dy$	$\sqrt{4+\sqrt{9+\sqrt{x}}}$	dx, $x > 0$	
and $y(0) = 1$	$\sqrt{7}$, then $y(256) =$	Manager Halle in		
[A] 80	[B] 9	[C] 16	LID 3	
	nly chosen nonnegative int 10. Then the probability the		found to satisfy the eq	uation 11
[A] ⁵ / ₁₁	[B] $\frac{1}{2}$	[C] 6/11	[D] 36/55	
可由	Page for Space f	r rough work	一人多	26
	+ 22+82752 +	93412412	=5	
Fx 39	7=3673	(Ly Ex	19+VA F1X9+	原)=9
2 5 3 3 c/x		94 FZ =		
3(3c, ×3c2×		から	= 2+ St	
The - Pary	12±9	生二级人口	+E	
2 Jt+4 =	24			

SECTION 2 (Maximum Marks: 28)

- . This section contains SEVEN questions
- Each question has FOUR options [A], [B], [C] and [D]. ONE OR MORE THAN ONE of
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS
- · For each question, marks will be awarded in one of the following categories:

Full Marks If only the bubble(s) corresponding to all the correct option(s) : +4

is(are) darkened

Partial Marks : +1 For darkening a bubble corresponding to each correct option,

provided NO incorrect option is darkened

If none of the bubbles is darkened Zero Marks

Negative Marks : -2 In all other cases

• For example, if [A], [C] and [D] are all the correct options for a question, darkening all these three will get +4 marks; darkening only [A] and [D] will get +2 marks; and darkening [A] and [B] will get -2 marks, as a wrong option is also darkened

Q.44 If the line $x = \alpha$ divides the area of region $R = \{(x, y) \in \mathbb{R}^2 : x^3 \le y \le x, 0 \le x \le 1\}$ into two equal parts, then

[A]
$$0 < \alpha \le \frac{1}{2}$$

[C]
$$\alpha^4 + 4\alpha^2 - 1 = 0$$

[B] $2\alpha^4 - 4\alpha^2 + 1 = 0$

$$|D|^{\frac{1}{2}} < \alpha < 1$$

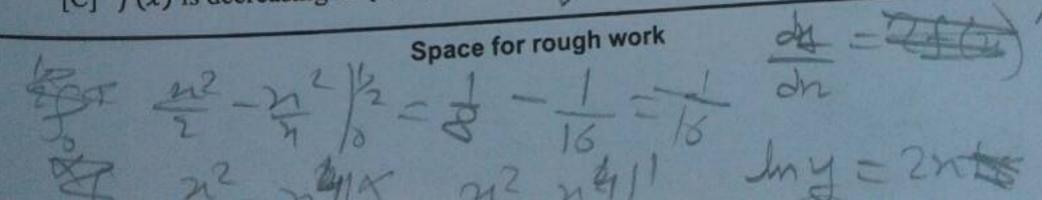
Q.45 If $f: \mathbb{R} \to \mathbb{R}$ is a differentiable function such that f'(x) > 2f(x) for all $x \in \mathbb{R}$, and f(0) = 1, then

[A]
$$f(x) > e^{2x}$$
 in $(0, \infty)$

[B]
$$f'(x) < e^{2x} \text{ in } (0, \infty)$$

[C] f(x) is decreasing in $(0, \infty)$

[D] f(x) is increasing in $(0, \infty)$



1-22+22

Q.46 Let
$$f(x) = \frac{1-x(1+|1-x|)}{|1-x|} \cos\left(\frac{1}{1-x}\right)$$
 for $x \neq 1$. Then

[A]
$$\lim_{x\to 1^+} f(x)$$
 does not exist

[B]
$$\lim_{x\to 1^-} f(x)$$
 does not exist

[C]
$$\lim_{x\to 1^+} f(x) = 0$$

[D]
$$\lim_{x\to 1^-} f(x) = 0$$

Q.47 If
$$g(x) = \int_{\sin x}^{\sin(2x)} \sin^{-1}(t) dt$$
, then

$$Ag'\left(\frac{\pi}{2}\right) = -2\pi$$

[B]
$$g'\left(\frac{\pi}{2}\right) = 2\pi$$

[C]
$$g'\left(-\frac{\pi}{2}\right) = 2\pi$$

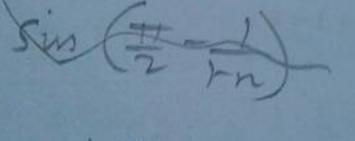
$$\int g'\left(-\frac{\pi}{2}\right) = -2\pi$$

Q.48 If
$$I = \sum_{k=1}^{98} \int_{k}^{k+1} \frac{k+1}{x(x+1)} dx$$
, then

[A]
$$I > \frac{49}{50}$$

[B]
$$I < \frac{49}{50}$$

Space for rough work



2.303

- n cosn

(k+) [** + + 1 on?

$$[A] \text{ If } f(x) = \begin{vmatrix} \cos(2x) & \cos(2x) & \sin(2x) \\ -\cos x & \cos x & -\sin x \\ \sin x & \sin x & \cos x \end{vmatrix}, \text{ then }$$

$$[A] f'(x) = 0 \text{ at exactly three points in } (-\pi, \pi)$$

$$[B] f'(x) = 0 \text{ at more than three points in } (-\pi, \pi)$$

$$[C] f(x) \text{ attains its maximum at } x = 0$$

$$[D] f(x) \text{ attains its minimum at } x = 0$$

$$[D] f(x) \text{ attains its minimum at } x = 0$$

$$[A] \sqrt{3} \tan \frac{\alpha}{2} - \tan \frac{\beta}{2} = 0$$

$$[C] \sqrt{3} \tan \frac{\alpha}{2} + \tan \frac{\beta}{2} = 0$$

$$[D] \tan \frac{\alpha}{2} - \sqrt{3} \tan \frac{\beta}{2} = 0$$

$$[D] \tan \frac{\alpha}{2} - \sqrt{3} \tan \frac{\beta}{2} = 0$$

$$[D] \tan \frac{\alpha}{2} - \sqrt{3} \tan \frac{\beta}{2} = 0$$

$$[D] \tan \frac{\alpha}{2} - \sqrt{3} \tan \frac{\beta}{2} = 0$$

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$$[D] \tan \frac{\alpha}{2} - \sqrt{3} \tan \frac{\beta}{2} = 0$$

$$[D] \tan \frac{\alpha}{2} - \sqrt{3} \tan \frac{\beta}{2} = 0$$

 $(n) = \frac{1}{2} - 2s^2 - 2s^2 \cdot 2e^2$ $(n) = \frac{1}{2} - 2s^2 - 2s^2 \cdot 2e^2$ $(n) = \frac{1}{2} - 2s^2 - 2s^2 \cdot 2e^2$ $\frac{1}{2} - 2s^2 - 2s^2 \cdot 2e^2$ $\frac{1}{2} - 2s^2 - 2s^2 \cdot 2e^2$ $\frac{1}{2} - 2s^2 - 4s^2 - 2s^2 \cdot 2e^2$ $\frac{1}{2} - 2s^2 - 4s^2 - 2s^2 \cdot 2e^2$ $\frac{1}{2} - 2s^2 - 4s^2 - 2s^2 \cdot 2e^2$ $\frac{1}{2} - 2s^2 - 4s^2 - 2s^2 \cdot 2e^2$ $\frac{1}{2} - 2s^2 - 4s^2 - 2s^2 \cdot 2e^2$ $\frac{1}{2} - 2s^2 - 4s^2 - 2s^2 \cdot 2e^2$ $\frac{1}{2} - 2s^2 - 4s^2 - 2s^2 \cdot 2e^2$ $\frac{1}{2} - 2s^2 - 4s^2 - 2s^2 \cdot 2e^2$ $\frac{1}{2} - 2s^2 - 4s^2 - 2s^2 \cdot 2e^2$ $\frac{1}{2} - 2s^2 - 4s^2 - 2s^2 \cdot 2e^2$ $\frac{1}{2} - 2s^2 - 4s^2 - 2s^2 \cdot 2e^2$ $\frac{1}{2} - 2s^2 - 4s^2 - 2s^2 \cdot 2e^2$ $\frac{1}{2} - 2s^2 - 4s^2 - 2s^2 \cdot 2e^2$ $\frac{1}{2} - 2s^2 - 4s^2 - 2s^2 \cdot 2e^2$ $\frac{1}{2} - 2s^2 - 4s^2 - 2s^2 \cdot 2e^2$ $\frac{1}{2} - 2s^2 - 4s^2 - 2s^2 \cdot 2e^2$ $\frac{1}{2} - 2s^2 - 4s^2 - 2s^2 \cdot 2e^2$ $\frac{1}{2} - 2s^2 - 4s^2 - 2s^2 \cdot 2e^2$ $\frac{1}{2} - 2s^2 - 4s^2 - 2s^2 \cdot 2e^2$ $\frac{1}{2} - 2s^2 - 4s^2 - 2s^2 \cdot 2e^2$ $\frac{1}{2} - 2s^2 - 4s^2 - 2s^2 \cdot 2e^2$ $\frac{1}{2} - 2s^2 - 4s^2 - 2s^2 \cdot 2e^2$ $\frac{1}{2} - 2s^2 - 4s^2 - 2s^2 \cdot 2e^2$ $\frac{1}{2} - 2s^2 - 4s^2 - 2s^2 \cdot 2e^2$ $\frac{1}{2} - 2s^2 - 4s^2 - 2s^2 \cdot 2e^2$ $\frac{1}{2} - 2s^2 - 4s^2 - 2s^2 \cdot 2e^2$ $\frac{1}{2} - 2s^2 - 4s^2 - 2s^2 \cdot 2e^2$ $\frac{1}{2} - 2s^2 - 4s^2 - 2s^2 \cdot 2e^2$ $\frac{1}{2} - 2s^2 - 4s^2 - 2s^2 \cdot 2e^2$ $\frac{1}{2} - 2s^2 - 4s^2 - 2s^2 \cdot 2e^2$ $\frac{1}{2} - 2s^2 - 4s^2 - 2s^2 \cdot 2e^2$ $\frac{1}{2} - 2s^2 - 4s^2 - 2s^2 \cdot 2e^2$ $\frac{1}{2} - 2s^2 - 2s^2 - 2s^2 \cdot 2e^2$ $\frac{1}{2} - 2s^2 - 2s^2 - 2s^2 \cdot 2e^2$ $\frac{1}{2} - 2s^2 - 2s^2 - 2s^2 \cdot 2e^2$ $\frac{1}{2} - 2s^2 - 2s^2 - 2s^2 \cdot 2e^2$ $\frac{1}{2} - 2s^2 - 2s^2 - 2s^2 \cdot 2e^2$ $\frac{1}{2} - 2s^2 - 2s^2 - 2s^2 \cdot 2e^2$ $\frac{1}{2} - 2s^2 - 2s^2 - 2s^2 \cdot 2e^2$ $\frac{1}{2} - 2s^2 - 2s^2 - 2s^2 \cdot 2e^2$ $\frac{1}{2} - 2s^2 - 2s^2 - 2s^2 \cdot 2e^2$ $\frac{1}{2} - 2s^2 - 2s^2 - 2s^2 \cdot 2e^2$ $\frac{1}{2} - 2s^2 - 2s^2 - 2s^2 \cdot 2e^2$ $\frac{1}{2} - 2s^2 - 2s^2 - 2s^2 \cdot 2e^2$

SECTION 3 (Maximum Marks: 12)

- · This section contains TWO paragraphs
- Based on each paragraph, there are TWO questions
- · Each question has FOUR options [A], [B], [C], and [D]. ONLY ONE of these four options is correct
- For each question, darken the bubble corresponding to the correct option in the ORS
- For each question, marks will be awarded in one of the following categories:

If only the bubble corresponding to the correct option is darkened Full Marks

Zero Marks In all other cases

PARAGRAPH 1

Let O be the origin, and \overrightarrow{OX} , \overrightarrow{OY} , \overrightarrow{OZ} be three unit vectors in the directions of the sides \overrightarrow{QR} , \overrightarrow{RP} , \overrightarrow{PQ} , respectively, of a triangle PQR.

 $Q.51 \mid \overrightarrow{OX} \times \overrightarrow{OY} \mid =$

[A]
$$\sin(Q + R)$$
 [B] $\sin 2R$

[B]
$$\sin 2R$$

[C]
$$\sin(P+R)$$

[C]
$$\sin(P+R)$$
 \[D] $\sin(P+Q)$

Q.52 If the triangle PQR varies, then the minimum value of

$$cos(P+Q) + cos(Q+R) + cos(R+P)$$

is

[A]
$$-\frac{5}{3}$$

$$\left| \frac{3}{2} \right|$$

$$[C] \frac{5}{3}$$

$$[D] \frac{3}{2}$$

PARAGRAPH 2

Let p, q be integers and let α , β be the roots of the equation, $x^2 - x - 1 = 0$, where $\alpha \neq \beta$. For n = 0, 1, 2, ..., let $a_n = p\alpha^n + q\beta^n$.

FACT: If a and b are rational numbers and $a + b\sqrt{5} = 0$, then a = 0 = b.

Q.53 If $a_4 = 28$, then p + 2q =

[A] 12

[B] 14

[C] 21

[D] 7

[A] $2a_{11} + a_{10}$ [B] $a_{11} - a_{10}$ [C] $a_{11} + 2a_{10}$ [D] $a_{11} + a_{10}$

END OF THE QUESTION PAPER

Space for rough work ~= 1 to 5, B= 1- 5 p 556+24-8 7 \$ \frac{7}{2} \fra