

**MA**

**Mathematics**

**Calculus:** Functions of two or more variables, continuity, directional derivatives, partial derivatives, total derivative, maxima and minima, saddle point, method of Lagrange's multipliers; Double and Triple integrals and their applications to area, volume and surface area; Vector Calculus: gradient, divergence and curl, Line integrals and Surface integrals, Green's theorem, Stokes' theorem, and Gauss divergence theorem.

**Linear Algebra:** Finite dimensional vector spaces over real or complex fields; Linear transformations and their matrix representations, rank and nullity; systems of linear equations, characteristic polynomial, eigen values and eigen vectors, diagonalization, minimal polynomial, Cayley-Hamilton Theorem, Finite dimensional inner product spaces, Gram-Schmidt orthonormalization process, symmetric, skew-symmetric, Hermitian, skew-Hermitian, normal, orthogonal and unitary matrices; diagonalization by a unitary matrix, Jordan canonical form; bilinear and quadratic forms.

**Real Analysis:** Metric spaces, connectedness, compactness, completeness; Sequences and series of functions, uniform convergence, Ascoli-Arzelà theorem; Weierstrass approximation theorem; contraction mapping principle, Power series; Differentiation of functions of several variables, Inverse and Implicit function theorems; Lebesgue measure on the real line, measurable functions; Lebesgue integral, Fatou's lemma, monotone convergence theorem, dominated convergence theorem.

**Complex Analysis:** Functions of a complex variable: continuity, differentiability, analytic functions, harmonic functions; Complex integration: Cauchy's integral theorem and formula; Liouville's theorem, maximum modulus principle, Morera's theorem; zeros and singularities; Power series, radius of convergence, Taylor's series and Laurent's series; Residue theorem and applications for evaluating real integrals; Rouché's theorem, Argument principle, Schwarz lemma; Conformal mappings, Möbius transformations.

**Ordinary Differential equations:** First order ordinary differential equations, existence and uniqueness theorems for initial value problems, linear ordinary differential equations of higher order with constant coefficients; Second order linear ordinary differential equations with variable coefficients; Cauchy-Euler equation, method of Laplace transforms for solving ordinary differential equations, series solutions (power series, Frobenius method); Legendre and Bessel functions and their orthogonal properties; Systems of linear first order ordinary differential equations, Sturm's oscillation and separation theorems, Sturm-Liouville eigenvalue problems, Planar autonomous systems of ordinary differential equations: Stability of stationary points for linear systems with constant coefficients, Linearized stability, Lyapunov functions.

**Algebra:** Groups, subgroups, normal subgroups, quotient groups, homomorphisms, automorphisms; cyclic groups, permutation groups, Group action, Sylow's theorems and their applications; Rings, ideals, prime and maximal ideals, quotient rings, unique factorization domains, Principle ideal domains, Euclidean domains, polynomial rings, Eisenstein's irreducibility criterion; Fields, finite fields, field extensions, algebraic extensions, algebraically closed fields

**Functional Analysis:** Normed linear spaces, Banach spaces, Hahn-Banach theorem, open mapping and closed graph theorems, principle of uniform boundedness; Inner-product

spaces, Hilbert spaces, orthonormal bases, projection theorem, Riesz representation theorem, spectral theorem for compact self-adjoint operators.

**Numerical Analysis:** Systems of linear equations: Direct methods (Gaussian elimination, LU decomposition, Cholesky factorization), Iterative methods (Gauss-Seidel and Jacobi) and their convergence for diagonally dominant coefficient matrices; Numerical solutions of nonlinear equations: bisection method, secant method, Newton-Raphson method, fixed point iteration; Interpolation: Lagrange and Newton forms of interpolating polynomial, Error in polynomial interpolation of a function; Numerical differentiation and error, Numerical integration: Trapezoidal and Simpson rules, Newton-Cotes integration formulas, composite rules, mathematical errors involved in numerical integration formulae; Numerical solution of initial value problems for ordinary differential equations: Methods of Euler, Runge-Kutta method of order 2.

**Partial Differential Equations:** Method of characteristics for first order linear and quasilinear partial differential equations; Second order partial differential equations in two independent variables: classification and canonical forms, method of separation of variables for Laplace equation in Cartesian and polar coordinates, heat and wave equations in one space variable; Wave equation: Cauchy problem and d'Alembert formula, domains of dependence and influence, non-homogeneous wave equation; Heat equation: Cauchy problem; Laplace and Fourier transform methods.

**Topology:** Basic concepts of topology, bases, subbases, subspace topology, order topology, product topology, quotient topology, metric topology, connectedness, compactness, countability and separation axioms, Urysohn's Lemma.

**Linear Programming:** Linear programming models, convex sets, extreme points; Basic feasible solution, graphical method, simplex method, two phase methods, revised simplex method; Infeasible and unbounded linear programming models, alternate optima; Duality theory, weak duality and strong duality; Balanced and unbalanced transportation problems, Initial basic feasible solution of balanced transportation problems (least cost method, north-west corner rule, Vogel's approximation method); Optimal solution, modified distribution method; Solving assignment problems, Hungarian method.



**GATE New Test Paper on (DA) Data Science and Artificial Intelligence**

**Syllabus**

**Probability and Statistics:** Counting (permutation and combinations), probability axioms, Sample space, events, independent events, mutually exclusive events, marginal, conditional and joint probability, Bayes Theorem, conditional expectation and variance, mean, median, mode and standard deviation, correlation, and covariance, random variables, discrete random variables and probability mass functions, uniform, Bernoulli, binomial distribution, Continuous random variables and probability distribution function, uniform, exponential, Poisson, normal, standard normal, t-distribution, chi-squared distributions, cumulative distribution function, Conditional PDF, Central limit theorem, confidence interval, z-test, t-test, chi-squared test.

**Linear Algebra:** Vector space, subspaces, linear dependence and independence of vectors, matrices, projection matrix, orthogonal matrix, idempotent matrix, partition matrix and their properties, quadratic forms, systems of linear equations and solutions; Gaussian elimination, eigenvalues and eigenvectors, determinant, rank, nullity, projections, LU decomposition, singular value decomposition.

**Calculus and Optimization:** Functions of a single variable, limit, continuity and differentiability, Taylor series, maxima and minima, optimization involving a single variable.

**Programming, Data Structures and Algorithms:** Programming in Python, basic data structures: stacks, queues, linked lists, trees, hash tables; Search algorithms: linear search and binary search, basic sorting algorithms: selection sort, bubble sort and insertion sort; divide and conquer: mergesort, quicksort; introduction to graph theory; basic graph algorithms: traversals and shortest path.

**Database Management and Warehousing:** ER-model, relational model: relational algebra, tuple calculus, SQL, integrity constraints, normal form, file organization, indexing, data types, data transformation such as normalization, discretization, sampling, compression; data warehouse modelling: schema for multidimensional data models, concept hierarchies, measures: categorization and computations.

**Machine Learning:** (i) Supervised Learning: regression and classification problems, simple linear regression, multiple linear regression, ridge regression, logistic regression, k-nearest neighbour, naive Bayes classifier, linear discriminant analysis, support vector machine, decision trees, bias-variance trade-off, cross-validation methods such as leave-one-out (LOO) cross-validation, k-folds cross-validation, multi-layer perceptron, feed-forward neural network; (ii) Unsupervised Learning: clustering algorithms, k-means/k-medoid, hierarchical clustering, top-down, bottom-up: single-linkage, multiple-linkage, dimensionality reduction, principal component analysis.

**AI:** Search: informed, uninformed, adversarial; logic, propositional, predicate; reasoning under uncertainty topics - conditional independence representation, exact inference through variable elimination, and approximate inference through sampling.

## Section 1: Physical Chemistry

**Structure:** Postulates of quantum mechanics. Operators. Time dependent and time independent Schrödinger equations. Born interpretation. Dirac bra-ket notation. Particle in a box: infinite and finite square wells; concept of tunnelling; particle in 1D, 2D and 3D-box; applications. Harmonic oscillator: harmonic and anharmonic potentials; hermite polynomials. Rotational motion: Angular momentum operators, Rigid rotor. Hydrogen and hydrogen-like atoms: atomic orbitals; radial distribution function. Multi-electron atoms: orbital approximation; electron spin; Pauli exclusion principle; Slater determinants. Approximation Methods: Variation method and secular determinants; first order perturbation techniques. Atomic units. Molecular structure and Chemical bonding: Born-Oppenheimer approximation; Valence bond theory and linear combination of atomic orbitals – molecular orbital (LCAO-MO) theory. Hybrid orbitals. Applications of LCAO-MO theory to  $H_2^+$ ,  $H_2$ ; orbital theory (MOT) of homo- and heteronuclear diatomic molecules. Hückel approximation and its application to annular  $\pi$  – electron systems.

**Group theory:** Symmetry elements and operations; Point groups and character tables; Internal coordinates and vibrational modes; symmetry adapted linear combination of atomic orbitals (LCAO-MO); construction of hybrid orbitals using symmetry aspects.

**Spectroscopy:** Atomic spectroscopy; Russell-Saunders coupling; Term symbols and spectral details; origin of selection rules. Rotational, vibrational, electronic and Raman spectroscopy of diatomic and polyatomic molecules. Line broadening. Einstein's coefficients. Relationship of transition moment integral with molar extinction coefficient and oscillator strength. Basic principles of nuclear magnetic resonance: gyromagnetic ratio; chemical shift, nuclear coupling.

**Equilibrium:** Laws of thermodynamics. Standard states. Thermochemistry. Thermodynamic functions and their relationships: Gibbs-Helmholtz and Maxwell relations, Gibbs-Duhem equation, van't Hoff equation. Criteria of spontaneity and equilibrium. Absolute entropy. Partial molar quantities. Thermodynamics of mixing. Chemical potential. Fugacity, activity and activity coefficients. Ideal and Non-ideal solutions, Raoult's Law and Henry's Law, Chemical equilibria. Dependence of equilibrium constant on temperature and pressure. Ionic mobility and conductivity. Debye-Hückel limiting law. Debye-Hückel-Onsager equation. Standard electrode potentials and electrochemical cells. Nernst Equation and its application, relationship between Electrode potential and thermodynamic quantities, Potentiometric and conduct metric titrations. Phase rule. Clausius- Clapeyron equation. Phase diagram of one component systems:  $CO_2$ ,  $H_2O$ , S; two component systems: liquid- vapour, liquid-liquid and solid-liquid systems. Fractional distillation. Azeotropes and eutectics. Statistical thermodynamics: micro canonical, canonical and grand canonical ensembles, Boltzmann distribution, partition functions and thermodynamic properties.

**Kinetics (Topic has been rearranged):** Elementary, parallel, opposing and consecutive reactions. Steady state approximation. Mechanisms of complex reactions. Unimolecular reactions. Potential energy surfaces and classical trajectories, Concept of Saddle points, Transition state theory: Eyring equation, thermodynamic aspects. Kinetics of polymerization. Catalysis concepts and enzyme catalysis. Kinetic isotope effects. Fast reaction kinetics: relaxation and flow methods. Diffusion controlled reactions. Kinetics of photochemical and photo physical processes.

**Surfaces and Interfaces:** Physisorption and chemisorption. Langmuir, Freundlich and Brunauer–Emmett–Teller (BET) isotherms. Surface catalysis: Langmuir-Hinshelwood mechanism. Surface tension, viscosity. Self-assembly. Physical chemistry of colloids, micelles and macromolecules.

## Section 2: Inorganic Chemistry

**Main Group Elements:** Hydrides, halides, oxides, oxoacids, nitrides, sulfides – shapes and reactivity. Structure and bonding of boranes, carboranes, silicones, silicates, boron nitride, borazines and phosphazenes. Allotropes of carbon, phosphorous and sulphur. Industrial synthesis of compounds of main group elements. Chemistry of noble gases, pseudohalogens, and interhalogen compounds. Acid-base concepts and principles (Lewis, Brønsted, HSAB and acid-base catalysis).

**Transition Elements:** Coordination chemistry – structure and isomerism, theories of bonding (VBT, CFT, and MOT). Energy level diagrams in various crystal fields, CFSE, applications of CFT, Jahn-Teller distortion. Electronic spectra of transition metal complexes: spectroscopic term symbols, selection rules, Orgel and Tanabe-Sugano diagrams, nephelauxetic effect and Racah parameter, charge-transfer spectra. Magnetic properties of transition metal complexes. Ray-Dutt and Bailar twists, Reaction mechanisms: kinetic and thermodynamic stability, substitution and redox reactions. Metal-metal multiple bond.

**Lanthanides and Actinides:** Recovery. Periodic properties, spectra and magnetic properties.

**Organometallics:** 18-Electron rule; metal-alkyl, metal-carbonyl, metal-olefin and metal-carbene complexes and metallocenes. Fluxionality in organometallic complexes. Types of organometallic reactions. Homogeneous catalysis - Hydrogenation, hydroformylation, acetic acid synthesis, metathesis and olefin oxidation. Heterogeneous catalysis - Fischer-Tropsch reaction, Ziegler-Natta polymerization.

**Radioactivity:** Detection of radioactivity, Decay processes, half-life of radioactive elements, fission and fusion processes.

**Bioinorganic Chemistry:** Ion ( $\text{Na}^+$  and  $\text{K}^+$ ) transport, oxygen binding, transport and utilization, electron transfer reactions, nitrogen fixation, metalloenzymes containing magnesium, molybdenum, iron, cobalt, copper and zinc.

**Solids:** Crystal systems and lattices, Miller planes, crystal packing, crystal defects, Bragg's law, ionic crystals, structures of  $\text{AX}$ ,  $\text{AX}_2$ ,  $\text{ABX}_3$  type compounds, spinels, band theory, metals and semiconductors.

**Instrumental Methods of Analysis:** UV-visible, fluorescence and FTIR spectrophotometry, NMR and ESR spectroscopy, mass spectrometry, atomic absorption spectroscopy, Mössbauer spectroscopy (Fe and Sn) and X-ray crystallography. Chromatography including GC and HPLC. Electroanalytical methods- polarography, cyclic voltammetry, ion-selective electrodes. Thermoanalytical methods.

### Section 3: Organic Chemistry

**Stereochemistry:** Chirality and symmetry of organic molecules with or without chiral centres and determination of their absolute configurations. Relative stereochemistry in compounds having more than one stereogenic centre. Homotopic, enantiotopic and diastereotopic atoms, groups and faces. Stereoselective and stereospecific synthesis. Conformational analysis of acyclic and cyclic compounds. Geometrical isomerism and optical isomerism. Configurational and conformational effects, atropisomerism, and neighbouring group participation on reactivity and selectivity/specificity.

**Reaction Mechanisms:** Basic mechanistic concepts – kinetic versus thermodynamic control, Hammond's postulate and Curtin-Hammett principle. Methods of determining reaction mechanisms through kinetics, identification of products, intermediates and isotopic labelling. Linear free-energy relationship – Hammett and Taft equations. Nucleophilic and electrophilic substitution reactions (both aromatic and aliphatic). Addition reactions to carbon-carbon and carbon-heteroatom (N and O) multiple bonds. Elimination reactions. Reactive intermediates – carbocations, carbanions, carbenes, nitrenes, arynes and free radicals. Molecular rearrangements.

**Organic Synthesis:** Synthesis, reactions, mechanisms and selectivity involving the following classes of compounds – alkenes, alkynes, arenes, alcohols, phenols, aldehydes, ketones, carboxylic acids, esters, nitriles, halides, nitro compounds, amines and amides. Uses of Mg, Li, Cu, B, Zn, P, S, Sn and Si based reagents in organic synthesis. Carbon-carbon bond formation through coupling reactions - Heck, Suzuki, Stille, Sonogoshira, Negishi, Kumada, Hiyama, Tsuji-Trost, olefin metathesis and McMurry. Concepts of multistep synthesis - retrosynthetic analysis, strategic disconnections, synthons and synthetic equivalents. Atom economy and Green Chemistry, Umpolung reactivity – formyl and acyl anion equivalents. Selectivity in organic synthesis – chemo-, regio- and stereoselectivity. Protection and deprotection of functional groups. Concepts of asymmetric synthesis – resolution (including enzymatic), desymmetrization and use of chiral auxiliaries, organocatalysis. Carbon-carbon and carbon-heteroatom bond forming reactions through enolates (including boron enolates), enamines and silyl enol ethers. Stereoselective addition to C=O groups (Cram, Prelog and Felkin-Anh models).

**Pericyclic Reactions and Photochemistry:** Electrocyclic, cycloaddition and sigmatropic reactions. Orbital correlations - FMO and PMO treatments, Woodward-Hoffmann rule. Photochemistry of alkenes, arenes and carbonyl compounds. Photooxidation and photoreduction. Di- $\pi$ -methane rearrangement, Barton-McCombie reaction, Norrish type-I and II cleavage reaction.

**Heterocyclic Compounds:** Structure, preparation, properties and reactions of furan, pyrrole, thiophene, pyridine, indole, quinoline and isoquinoline.

**Biomolecules:** Structure, properties and reactions of mono- and di-saccharides, physicochemical properties of amino acids, chemical synthesis of peptides, chemical structure determination of peptides and proteins, structural features of proteins, nucleic acids, lipids, steroids, terpenoids, carotenoids, and alkaloids.

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**Experimental techniques in organic chemistry:** Optical rotation (polarimetry). Applications of various chromatographic techniques such as thin-layer, column, HPLC and GC. Applications of UV-visible, IR, NMR and Mass spectrometry in the structural determination of organic molecules.



**PH**

**PHYSICS**

### Section 1: Mathematical Physics

**Vector calculus:** linear vector space: basis, orthogonality and completeness; matrices; similarity transformations, diagonalization, eigen values and eigen vectors; linear differential equations: second order linear differential equations and solutions involving special functions; complex analysis: Cauchy-Riemann conditions, Cauchy's theorem, singularities, residue theorem and applications; Laplace transform, Fourier analysis; elementary ideas about tensors: covariant and contravariant tensors.

### Section 2: Classical Mechanics

**Lagrangian formulation:** D'Alembert's principle, Euler-Lagrange equation, Hamilton's principle, calculus of variations; symmetry and conservation laws; central force motion: Kepler problem and Rutherford scattering; small oscillations: coupled oscillations and normal modes; rigid body dynamics: inertia tensor, orthogonal transformations, Euler angles, Torque free motion of a symmetric top; Hamiltonian and Hamilton's equations of motion; Liouville's theorem; canonical transformations: action-angle variables, Poisson brackets, Hamilton-Jacobi equation.

**Special theory of relativity:** Lorentz transformations, relativistic kinematics, mass-energy equivalence.

### Section 3: Electromagnetic Theory

Solutions of electrostatic and magnetostatic problems including boundary value problems; method of images; separation of variables; dielectrics and conductors; magnetic materials; multipole expansion; Maxwell's equations; scalar and vector potentials; Coulomb and Lorentz gauges; electromagnetic waves in free space, non-conducting and conducting media; reflection and transmission at normal and oblique incidences; polarization of electromagnetic waves; Poynting vector, Poynting theorem, energy and momentum of electromagnetic waves; radiation from a moving charge.

### Section 4: Quantum Mechanics

Postulates of quantum mechanics; uncertainty principle; Schrodinger equation; Dirac Bra-Ket notation, linear vectors and operators in Hilbert space; one dimensional potentials: step potential, finite rectangular well, tunneling from a potential barrier, particle in a box, harmonic oscillator; two and three dimensional systems: concept of degeneracy; hydrogen atom; angular momentum and spin; addition of angular momenta; variational method and WKB approximation, time independent perturbation theory; elementary scattering theory, Born approximation; symmetries in quantum mechanical systems.

### Section 5: Thermodynamics and Statistical Physics

Laws of thermodynamics; macrostates and microstates; phase space; ensembles; partition function, free energy, calculation of thermodynamic quantities; classical and quantum



statistics; degenerate Fermi gas; black body radiation and Planck's distribution law; Bose-Einstein condensation; first and second order phase transitions, phase equilibria, critical point.

### **Section 6: Atomic and Molecular Physics**

Spectra of one-and many-electron atoms; spin-orbit interaction: LS and jj couplings; fine and hyperfine structures; Zeeman and Stark effects; electric dipole transitions and selection rules; rotational and vibrational spectra of diatomic molecules; electronic transitions in diatomic molecules, Franck-Condon principle; Raman effect; EPR, NMR, ESR, X-ray spectra; lasers: Einstein coefficients, population inversion, two and three level systems.

### **Section 7: Solid State Physics**

Elements of crystallography; diffraction methods for structure determination; bonding in solids; lattice vibrations and thermal properties of solids; free electron theory; band theory of solids: nearly free electron and tight binding models; metals, semiconductors and insulators; conductivity, mobility and effective mass; Optical properties of solids; Kramer's-Kronig relation, intra- and inter-band transitions; dielectric properties of solid; dielectric function, polarizability, ferroelectricity; magnetic properties of solids; dia, para, ferro, antiferro and ferri-magnetism, domains and magnetic anisotropy; superconductivity: Type-I and Type II superconductors, Meissner effect, London equation, BCS Theory, flux quantization.

### **Section 8: Electronics**

**Semiconductors in equilibrium:** electron and hole statistics in intrinsic and extrinsic semiconductors; metal-semiconductor junctions; Ohmic and rectifying contacts; PN diodes, bipolar junction transistors, field effect transistors; negative and positive feedback circuits; oscillators, operational amplifiers, active filters; basics of digital logic circuits, combinational and sequential circuits, flip-flops, timers, counters, registers, A/D and D/A conversion.

### **Section 9: Nuclear and Particle Physics**

Nuclear radii and charge distributions, nuclear binding energy, electric and magnetic moments; semi-empirical mass formula; nuclear models; liquid drop model, nuclear shell model; nuclear force and two nucleon problem; alpha decay, beta-decay, electromagnetic transitions in nuclei; Rutherford scattering, nuclear reactions, conservation laws; fission and fusion; particle accelerators and detectors; elementary particles; photons, baryons, mesons and leptons; quark model; conservation laws, isospin symmetry, charge conjugation, parity and time-reversal invariance.